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Universal Computing Machine for Number Genetics in the Proportiones Perfectus Theory

Lovemore Mamombe[1*]

[1]Independent Researcher, Harare, Zimbabwe.

Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

The *proportiones perfectus* law states that $\sigma_x^y = \frac{x+\sqrt{x^2+4y}}{2}$ is a *proportione perfectus* if $1 \le y \le x$ such that for an arbitrary positive integer h_1, there exists an integer sequence H_n defined simultaneously by the quasigeometric relation $h_{n+1} = round(\sigma_x^y h_n), n \ge 1$ and the arithmetic relation $h_{n+2} = xh_{n+1} + yh_n, n \ge 1$. When $x = y = 1$, σ_x^y is the golden mean. When $x = 2, y = 1, \sigma_x^y$ is the silver mean. In previous works we introduced the theory of number genetics – a framework of logic within which the golden section is studied. In this work we apply the concept to all *proportiones perfectus*. Let H_n be defined as above. Furthermore, let h_1 satisfy $\begin{cases} round\left(\frac{h_1}{\sigma_x^y}\right) = w \\ round(w\sigma_x^y) \ne h_1 \end{cases}$. Now let H_n' be H_n with $h_1 = 1$. Again let G_n be defined as H_n. A robust universal computing machine is herein developed for the purpose of establishing the relationship $h_i = g_{n+i-1} \pm h_i', i, n \ge 1$, which relationship is key to the logic protocol. It is clear therefore that this system of logic presents H_n' as the building block of every H_n (including itself) within a particular σ_x^y regime. Clearly number genetics takes central place in the proportiones perfectus theory.

**Corresponding author: E-mail: mamombel@gmail.com;*

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1 Introduction

Let

$$\sigma_x^y = \frac{x+\sqrt{x^2+4y}}{2} \tag{1.1}$$

σ_x^y is *a proportione perfectus* if $1 \le y \le x$. For an arbitrary positive integer h_1, there exists

$$H_n = h_1, h_2, xh_2 + yh_1, \dots \tag{1.2}$$

such that

$$h_{n+1} = round(\sigma_x^y h_n), n \ge 1 \tag{1.3}$$

But in practice we do not need arbitrary h_1. Let h_1 satisfy

$$\left.\begin{aligned} round\left(\frac{h_1}{\sigma_x^y}\right) = w \\ round(w\sigma_x^y) \ne h_1 \end{aligned}\right\} \tag{1.4}$$

Let us have the base sequence for σ_x^y with $h_1 = 1$ designated $H_n^{'}$. Let $h_1 = a$; designate this sequence H_n. Let $h_1 = b$; giving rise to G_n. The object of number genetics is to establish the relationship

$$h_i = g_{n+i-1} \pm h_i^{'}, i, n \ge 1 \tag{1.5}$$

In [1] we studied number genetics for Equation (1.3) with $x = 1, y = 1$. In this work we develop a robust universal computing machine that establishes equation (1.5) for a given h_1 in a particular σ_x^y regime, $1 \le y \le x$.

2 Universal Computing Machine

Axiom 2.1

Let σ_x^y be a proportione perfectus. Let

$$\left.\begin{aligned} H_n^{'} = h_1^{'}, h_2^{'}, xh_2^{'} + yh_1^{'}, \dots, h_1^{'} = 1 \\ h_{n+1}^{'} = round(\sigma_x^y h_n^{'}), n \ge 1 \end{aligned}\right\} \tag{2.1}$$

If $y \le \frac{x}{2}$, then $h_2^{'} = x$. If $y > \frac{x}{2}$, then $h_2^{'} = x + 1$.

Lemma 2.1

Let σ_x^y be a proportione perfectus. Let equation (2.1) apply. Let $c_1^{'} = h_1^{'} h_3^{'} - (h_2^{'})^2$.
If $y \le \frac{x}{2}$, then $c_1^{'} = y$. If $y > \frac{x}{2}$, then $c_1^{'} = y - x - 1$.

Proof

For $y \leq \frac{x}{2}$*, from axiom 2.1,*

$$H_n^{'} = 1, x, x^2 + y, \dots \quad (2.2)$$

It follows $c_1^{'} = h_1^{'}h_3^{'} - (h_2^{'})^2 = 1(x^2 + y) - x^2 = y$.

For $y > \frac{x}{2}$*, from axiom 2.1,*

$$H_n^{'} = 1, x + 1, x^2 + x + y, \dots \quad (2.3)$$

It follows $c_1^{'} = h_1^{'}h_3^{'} - (h_2^{'})^2 = 1(x^2 + x + y) - x^2 - 2x - 1 = y - x - 1$.

Theorem 2.1

Let σ_x^y *be a proportione perfectus. Let equation (2.1) apply. Let* $c_1^{'} = h_1^{'}h_3^{'} - (h_2^{'})^2$ *be positive. Let*

$$\left.\begin{matrix} H_n = h_1, h_2, xh_2 + yh_1, h_1 \geq 1 \\ h_{n+1} = round\left(\sigma_x^y h_n\right), n \geq 1 \end{matrix}\right\} \quad (2.4)$$

$$h_2 - h_1 h_2^{'} = y g_{n-1} \quad (2.5)$$

such that

$$h_i = g_{n+i-1} \pm h_i^{'}, i, n \geq 1 \quad (2.6)$$

Proof

<u>*Scenario I:*</u> $H_n = g_n - h_1^{'}, g_{n+1} - h_2^{'}, g_{n+2} - h_3^{'}, \dots$

From axiom 2.1 and lemma 2.1, $h_2^{'} = x$*. It follows*

$$\left.\begin{matrix} h_1 = g_n - 1 \\ h_2 = g_{n+1} - x \end{matrix}\right\} \quad (2.7)$$

Therefore,

$$\begin{aligned} h_2 - h_1 h_2^{'} &= (g_{n+1} - x) - (g_n - 1)x \\ &= g_{n+1} - x - xg_n + x \\ &= g_{n+1} - xg_n \end{aligned} \quad (2.8)$$

But

$$g_{n+1} = xg_n + yg_{n-1} \quad (2.9)$$

Thus equation (2.8) becomes

$$xg_n + yg_{n-1} - xg_n = yg_{n-1} \quad (2.10)$$

<u>*Scenario II:*</u> $H_n = g_n + h_1^{'}, g_{n+1} + h_2^{'}, g_{n+2} + h_3^{'}, \dots$

From axiom 2.1 and lemma 2.1, $h_2^{'} = x$. *It follows*

$$\left.\begin{aligned} h_1 &= g_n + 1 \\ h_2 &= g_{n+1} + x \end{aligned}\right\} \tag{2.11}$$

Therefore,

$$\begin{aligned} h_2 - h_1 h_2^{'} &= (g_{n+1} + x) - (g_n + 1)x \\ &= g_{n+1} + x - xg_n - x \\ &= g_{n+1} - xg_n \end{aligned} \tag{2.12}$$

and from equation (2.9), equation (2.12) reduces to

$$xg_n + yg_{n-1} - xg_n = yg_{n-1} \tag{2.13}$$

Theorem 2.2

Let σ_x^y *be a proportione perfectus. Let equation (2.1) apply. Let* $c_1^{'} = h_1^{'} h_3^{'} - (h_2^{'})^2$ *be negative. Let equation (2.4) apply.*

$$h_1 h_2^{'} - h_2 = g_n - yg_{n-1} \tag{2.14}$$

such that equation (2.6) holds.

Proof

<u>*Scenario I:*</u> $H_n = g_n - h_1^{'}, g_{n+1} - h_2^{'}, g_{n+2} - h_3^{'}, \ldots$
From axiom 2.1 and lemma 2.1, $h_2^{'} = x + 1$. *It follows*

$$\left.\begin{aligned} h_1 &= g_n - 1 \\ h_2 &= g_{n+1} - x - 1 \end{aligned}\right\} \tag{2.15}$$

Therefore,

$$\begin{aligned} h_1 h_2^{'} - h_2 &= (g_n - 1)(x + 1) - (g_{n+1} - x - 1) \\ &= xg_n + g_n - g_{n+1} \\ &= xg_n + g_n - (xg_n + yg_{n-1}) \\ &= g_n - yg_{n-1} \end{aligned} \tag{2.16}$$

<u>*Scenario II:*</u> $H_n = g_n + h_1^{'}, g_{n+1} + h_2^{'}, g_{n+2} + h_3^{'}, \ldots$

From axiom 2.1 and lemma 2.1, $h_2^{'} = x + 1$. *It follows*

$$\left.\begin{aligned} h_1 &= g_n + 1 \\ h_2 &= g_{n+1} + x + 1 \end{aligned}\right\} \tag{2.17}$$

Therefore,

$$\begin{aligned} h_1 h_2^{'} - h_2 &= (g_n + 1)(x + 1) - (g_{n+1} + x + 1) \\ &= xg_n + g_n - g_{n+1} \\ &= xg_n + g_n - (xg_n + yg_{n-1}) \\ &= g_n - yg_{n-1} \end{aligned} \tag{2.18}$$

The universal computing machine in Fig. 2.1 is based on Lemma 2.1 and Theorems 2.1 and 2.2.

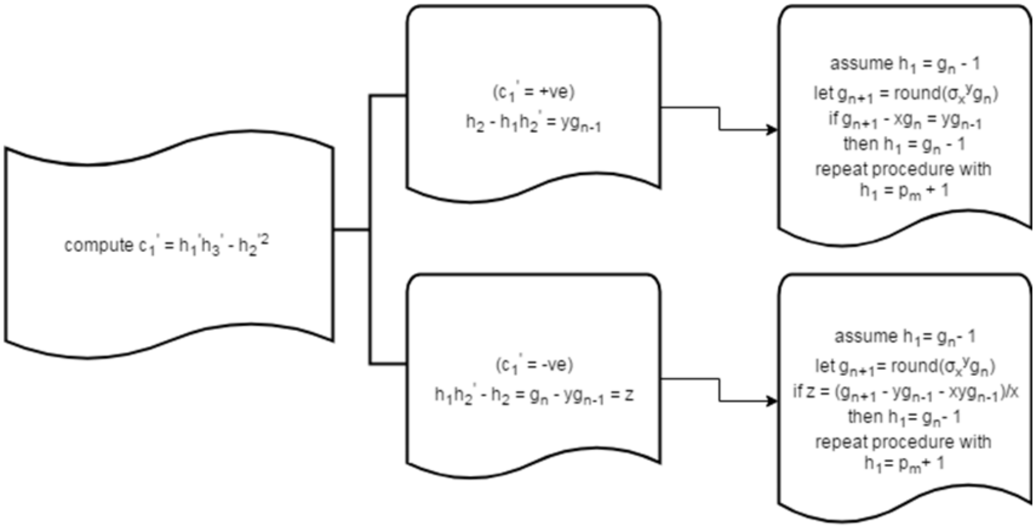

Fig. 2.1. Universal computing machine

3 Discussion: Bio-logical Paradoxes

3.1 Daughter cell from two mother cells

For illustration we use σ_2^2.

$$H_n^{'} = 1,3,8,22,60, \dots \tag{3.1}$$

Let

$$\left.\begin{aligned} P_n &= 6,16,44,120,328, \dots \\ Q_n &= 7,19,52,142,388, \dots \end{aligned}\right\} \tag{3.2}$$

Notice that both

$$q_i = h_{i+2}^{'} - h_i^{'}, i \geq 1 \tag{3.3}$$

and

$$q_i = p_i + h_i^{'}, i \geq 1 \tag{3.4}$$

hold.

Here, both $h_3^{'}$ and $p_1$claim q_1 as their daughter cell. Indeed the computing machine confirms that $h_3^{'}$ and p_1 are legitimate mother cells of q_1. For interest's sake let's assign gender (polarity) to these sequences. Let $h_i = g_{n+i-1} - h_i^{'}$ be *feminine* and $h_i = g_{n+i-1} + h_i^{'}$ be *masculine*. Equations (3.3) and (3.4) indicate that Q_n is both *feminine* and *masculine*. We must therefore appreciate the machine's calibration. The machine is calibrated to detect g_n such that equation (1.5) holds. But first of all, the machine detects that $h_i^{'} = g_{n+i-1} + h_i^{'}, i \geq 1$, where $G_n = 0,0,0,0, \dots$ is the null sequence. In some cases H_n' assembles itself from both the null sequence and the sequence $2(h_1', h_2', h_3', \dots)$, giving another example of the paradox discussed above. So the base sequence $H_n^{'}$ is the building block of every H_n, including itself.

3.2 Mother cell springing from daughter cell

For illustration we use σ_3^2.

$$H_n^{'} = 1,4,14,50,178,\ldots \quad (3.5)$$

Let

$$\left.\begin{aligned} P_n &= 2,7,25,89,317,\ldots \\ Q_n &= 3,11,39,139,495,\ldots \end{aligned}\right\} \quad (3.6)$$

Notice that

$$p_i = q_i - h_i^{'}, i \geq 1 \quad (3.7)$$

and

$$q_i = p_i + h_i^{'}, i \geq 1 \quad (3.8)$$

Equations (3.7) and (3.8) indicate that P_n and Q_n spring from each other. So p_1 is both mother cell and daughter cell of q_1 and vice-versa.

4 Application: Silver Pyramids

σ_2^1 is the silver mean and by definition, a *proportione perfectus*. Using this proportion, from relation (1.3) we assemble the first sequence

$$P_n = 1,2,5,12,29,\ldots \quad (4.1)$$

This sequence is well known in literature as the Pell sequence. The next few in this class, call them silver sequences, are:

$$\left.\begin{aligned} &3,7,17,41,99,\ldots \\ &4,10,24,58,140,\ldots \\ &6,14,34,82,198,\ldots \\ &8,19,46,111,268,\ldots \\ &9,22,53,128,309,\ldots \end{aligned}\right\} \quad (4.2)$$

The third sequence in equation (4.2) is the Pell-Lucas sequence.

Let H_n be an arbitrary silver sequence. To compute the *"mother cell"* of h_1, let

$$\left.\begin{aligned} h_2 - 2h_1 &= g_{n-1} \\ \sigma_2^1 g_{n-1} &= g_n \end{aligned}\right\} \quad (4.3)$$

such that

$$h_1 = g_n \pm 1 \quad (4.4)$$

and

$$h_i = g_{n+i-1} \pm p_i, i \geq 1, n \geq 2 \quad (4.5)$$

where $P_n = (4.1)$ and G_n is a silver sequence. This definition shall not be repeated hereinafter. From the computing machine presented in Section 2, we state the following four Axioms:

Axiom 4.1

Given an arbitrary Pell sequence H_n, $(h_1, h_2, \dots) \pm (p_1, p_2, \dots) \neq (q_1, q_2, \dots)$, Q_n is a Pell sequence.

Axiom 4.2

When a Pell sequence H_n is such that $h_i = g_{n+i-1} + p_i, i \geq 1, n \geq 2$, then
$(h_2, h_3, \dots) + (p_1, p_2, \dots) = (q_1, q_2, \dots)$, Q_n is a Pell sequence.

Axiom 4.3

When a Pell sequence H_n is such that $h_i = g_{n+i-1} - p_i, i \geq 1, n \geq 2$, then
$(h_2, h_3, \dots) - (p_1, p_2, \dots) = (q_1, q_2, \dots)$, Q_n is a Pell sequence.

Axiom 4.4

Given an arbitrary Pell sequence H_n, $(h_i, h_{i+1}, \dots) \pm (p_1, p_2, \dots) = (q_1, q_2, \dots), i \geq 3$, Q_n is a Pell sequence.

Axiom 4.4 shows the symmetry due to P_n that exists for $i \geq 3$ in every silver sequence.

4.1 Master silver pyramid

We construct a remarkable numerical structure, taking note of Axioms 4.1 to 4.4. If H_n is such that $h_1 = g_n - 1$, then h_1 is placed to the left hand side and in the same level with g_n. If H_n is such that $h_1 = g_n + 1$, then h_1 is placed to the right hand side and in the same level with g_n. All sequences are entered vertically. The master pyramid is that of P_n, given in Table 4.1.

Table 4.1: Master Silver Pyramid

				1							
				2	3						
			4	5	7	6	8				
16	11	9	10	12	17	14	19	13	15	18	20

We have given only the first four pyramid levels in table 4.1 due to space restrictions. The number and sum of elements per level are very important properties of the pyramid. Herein we are interested in the number of elements. We prove an important result:

Theorem 4.1

Let k_i be the number of elements in level i of the pyramid of P_n. For $i \geq 1$,

$$k_i = p_i \tag{4.6}$$

Proof

From axioms 4.1 to 4.4, p_1 does not produce any sequence, p_2 produces one sequence, and $p_i, i \geq 3$, produces two sequences. But these sequences produced also produce other sequences in the same manner as P_n. Take an arbitrary pyramid level i. Let the number of elements in this level be k_i. We need the constituent components of k_i.

Component I

Firstly we have k_{i-1} elements; guaranteed.

Component II

The k_{i-2} elements, from axiom 2.3, all produce two elements each, therefore we have $2k_{i-2}$ elements.

Component III

The elements that produce one element according to axioms 4.2 and 4.3 are those produced in the $(i-1)^{th}$ level, and these are given by $k_{i-1} - k_{i-2}$.

These three components constitute k_i. Therefore,

$$\begin{aligned} k_i &= k_{i-1} + 2k_{i-2} + k_{i-1} - k_{i-2} \\ &= 2k_{i-1} + k_{i-2} \end{aligned} \tag{4.7}$$

But equation (4.7) is subject to this constraint: $k_1 = 1, k_2 = 2$. It follows that $k_i = p_i$.
Proof is complete.

4.2 Other silver pyramids

The same construction procedure is followed as that of the master pyramid. From axioms 4.1 to 4.4, every silver sequence in principle behaves in the same manner as P_n with regard production of other sequences, therefore Corollary 4.1 arises from Theorem 4.1.

Corollary 4.1

Let H_n be an arbitrary Pell sequence. The number of elements in level i of the pyramid of H_n is given by p_i, where $P_n = (4.1)$.

Theorem 4.1 and Corollary 4.1 show that P_n models the number of elements in the levels of any silver pyramid, a law of mathematical beauty.

5 Conclusion

The universal computing machine developed herein applies to any arbitrary proportione perfectus. Since number genetics is the creation of a system of logic, it takes central place in the proportiones perfectus theory. Most striking is the number of elements in the pyramid levels of silver pyramids which follows the sequence $P_n = (4.1)$, thereby representing an important law of mathematical beauty. These silver pyramids, like the golden pyramids [2], have got important applications, the obvious ones being in communication.

The reader shall see [3-21] for some works on number theory and its applications.

Competing Interests

Author hereby declares that no competing interests exist in the publication of this work.

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