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Combined Optimized SFGM(1,1,k) Model and Its Application in Total Import and Export

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Authors' contributions

This work was carried out in collaboration between both authors. Author LY designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author LY managed the analyses of the study. Author XX managed the literature searches. Both authors read and approved the final manuscript.

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Method Article

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Abstract

Since the introduction of the fractional FGM(1,1) model, it has been widely used in all aspects of society. However, the original FGM(1,1) model has certain defects. Therefore, many experts and scholars have proposed many optimization methods for the shortcomings of the fractional FGM(1,1) model, and have been well applied. Since the gray action of the FGM(1,1) model is a fixed constant, the dynamic variation characteristics of the system cannot be well described. Based on this, this paper proposes a new SFGM(1,1,k) model by optimizing the gray action amount and reconstructing the background value of the optimized model using Simpson formula to improve the FGM(1,1) model. Then, apply it to the numerical examples of import and export totals. The results show that the combined optimization of SFGM(1,1,k) has higher fitting accuracy and better prediction effect. The validity and practicability of the SFGM(1,1,k) model are verified. The application range of FGM(1,1) model is extended.

Keywords: The total import and export; Simpson formula; gray action; background value; prediction accuracy.

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1 Introduction

Nowadays, with the rapid development of international trade, import and export trade plays an indispensable role in the national economy and plays a decisive role in the country's economic development. The total import and export volume is an important indicator used to measure the total size of a country's foreign trade, reflecting the total amount of goods entering and leaving a country's borders. In the past 2018, the total import and export of Chinese goods reached 30.5 trillion yuan, an increase of 9.7% from the previous year's historical high, once again creating a new historical record. Therefore, under the background that China's opening up to the outside world is deepening and foreign economic and trade influences on China's economic growth is increasingly prominent, it is particularly important to construct a reasonable and accurate forecasting model with certain innovation. An accurate and reasonable forecast will provide an important theoretical basis for China to optimize the structure of import and export trade, upgrade import and export trade, and make macroeconomic regulation and control of China's economic development.

It is precisely because of the important position of the total import and export in the development of the national economy, so a large number of scholars have invested in the forecasting work on the total import and export volume. Luo et al. [1] introduced the chaotic time series analysis method into the RBF neural network model, and applied it to the chaotic analysis and nonlinear prediction of the total time series of import and export, and achieved good prediction results. Huang [2] combined the gray system theory with the Markov transition probability to gray-scale the total import and export volume of China to ASEAN and the total import and export volume of the Guangxi Province to ASEAN, and establish metabolic GM(1,1) and residuals. The combined model of GM(1,1) and Markov is used for predictive analysis. Tian et al. [3] established the RBF neural network model to predict and analyze the total import and export volume of China. The application proved that the RBF neural network prediction effect is better than nonlinear regression and BP neural network prediction. Yang [4] conducted an empirical analysis of the total import and export volume of the Guangdong Province by combining the seasonal ARIMA model and outlier analysis. Subsequently, Li et al. [5] also used time series analysis to predict and analyze the total import and export volume of China, and concluded that the short-term prediction effect of ARMA model is better than long-term prediction. Ren [6] used the ARIMA model to analyze the total import and export of the Henan Province, and finally determined that the ARIMA (2,2,4) model has a good predictive effect. Xue et al. [7] used the econometric method to establish a forecasting model for the total import and export trade in the Jilin Province. Hu [8] used a multiple linear regression model to analyze the model and established a multi-factor predictive model. Recently, Wang et al. [9] used the time series method to analyze the total import and export volume of China, and established the corresponding seasonal time series SARIMA model.

It can be seen from the above research literature that the predictive analysis model for the total import and export mainly includes the time series analysis model, the BP neural network model, the RBF neural network model, the GM(1,1) model and combined prediction model. Since Professor Deng Julong founded the Grey System theory in 1978, the grey prediction model has been widely applied to economic, agricultural, military, and ecological fields [10-14].With the continuous improvement of the grey system theory, some scholars [15-16] have extended the integer order accumulation to the fractional order accumulation, and established a gray prediction model based on fractional order accumulation, which reduces the perturbation range of the grey prediction model solution. Greatly improved the prediction accuracy. In addition, on the basis of the classical grey system theory, a large number of scholars have continuously improved the model, and the accuracy of the gray prediction model has been continuously improved, and the model form has also changed. The main improvements include preprocessing the original data sequence [17-18], improving the initial conditions of the model [19-20], improving the background value of the model [21-25], and improving the parameters of the model [26-27]. Four aspects. Among them, many scholars [28-31] used a more accurate integral formula to transform the background value of the classical GM(1,1) model. There are also many scholars [32] to improve the accuracy of the model by improving the solution method or model form of parameters a, b, which has also been widely recognized by the academic community.

Based on the extensive research work done by the above scholars, this paper optimizes and improves on the basis of the fractional gray FGM(1,1) model. Considering the limitations of "single improvement" in the model, a combinatorial optimization method is proposed. The method considers the optimization of the gray action amount b and the background value, and uses the Chinese import and export total data to establish the FGM(1,1), GM(1,1) and SGM(1,1) models respectively. The optimized SFGM (1,1,k) model has higher prediction accuracy than the other three models.

2 Fractional FGM (1,1) Model

2.1 Fractional order accumulative operator and accumulative operator

Let the original sequence $X^{(0)}=\left(x^{(0)}(1),x^{(0)}(2),\ldots,x^{(0)}(n)\right)$ be defined, and the rth-order accumulation generating operator $(r\text{-}AGO)$ can be defined as follows.

Definition 1. Let sequence $X^{(r)}(r>0)$ be the accumulation generating operator $(r\text{-}AGO)$ of $X^{(0)}$, where

$$x^{(r)}(k)=\sum_{i=1}^{k}x^{(r-1)}(i),k=1,2,\ldots,n. \tag{1}$$

Expression $X^{(r)}=X^{(0)}A^{r}$ can be obtained from the matrix operation theory, where A^{r} denotes a $r\text{-}AGO$ matrix, and A^{r} can be expressed as follows

$$A^{r}=\begin{bmatrix} \begin{bmatrix} r\\0 \end{bmatrix} & \begin{bmatrix} r\\1 \end{bmatrix} & \begin{bmatrix} r\\2 \end{bmatrix} & \cdots & \begin{bmatrix} r\\n-1 \end{bmatrix} \\ 0 & \begin{bmatrix} r\\0 \end{bmatrix} & \begin{bmatrix} r\\1 \end{bmatrix} & \cdots & \begin{bmatrix} r\\n-2 \end{bmatrix} \\ 0 & 0 & \begin{bmatrix} r\\0 \end{bmatrix} & \cdots & \begin{bmatrix} r\\n-3 \end{bmatrix} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \begin{bmatrix} r\\0 \end{bmatrix} \end{bmatrix}_{n\times n}, \tag{2}$$

where

$$\begin{bmatrix} r\\n \end{bmatrix}=\begin{cases} 1 & n=0 \\ \dfrac{r(r+1)\cdots(r+n-1)}{n!} & n\in N^{+}. \end{cases} \tag{3}$$

In particular, when $r=1$, $x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i)$, gives $X^{(1)}=X^{(0)}A^{1}$, where A^{1} represents the 1-AGO matrix, which can be expressed as follows

$$A^{1}=\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & 1 & 1 & \cdots & 1 \\ 0 & 0 & 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}_{n\times n}. \tag{4}$$

Definition 2. Let $x^{(r-1)}(k)=\sum_{m=1}^{k}x^{(r-1)}(m)-\sum_{m=1}^{k-1}x^{(r-1)}(m)=x^{(r)}(k)-x^{(r)}(k-1), k=2,3,...,n$ be the gray rth-order subtraction sequence $(r\text{-}IAGO)$ .Similarly, the expression $X^{(0)}=X^{(r)}D^{r}$ can be obtained from the matrix operation theory, where $X^{(r)}=\left(x^{(r)}(1),x^{(r)}(2),x^{(r)}(3),\cdots,x^{(r)}(n)\right)$, D^{r} represents the $r\text{-}IAGO$ matrix, which can be expressed as follows

$$D^{r}=\begin{bmatrix}\begin{bmatrix}-r\\0\end{bmatrix} & \begin{bmatrix}-r\\1\end{bmatrix} & \begin{bmatrix}-r\\2\end{bmatrix} & \cdots & \begin{bmatrix}-r\\n-1\end{bmatrix}\\ 0 & \begin{bmatrix}-r\\0\end{bmatrix} & \begin{bmatrix}-r\\1\end{bmatrix} & \cdots & \begin{bmatrix}-r\\n-2\end{bmatrix}\\ 0 & 0 & \begin{bmatrix}-r\\0\end{bmatrix} & \cdots & \begin{bmatrix}-r\\n-3\end{bmatrix}\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & \begin{bmatrix}-r\\0\end{bmatrix}\end{bmatrix}_{n\times n}, \tag{5}$$

where

$$\begin{bmatrix}-r\\n\end{bmatrix}=\begin{cases}1 & n=0\\ \dfrac{(-r)(-r+1)\cdots(-r+n-1)}{n!} & n\in N^{+}.\end{cases} \tag{6}$$

In particular, when $r=1$, the 1-$IAGO$ sequence can be expressed as $x^{(0)}(k)=x^{(1)}(k)-x^{(1)}(k-1), k=2,3,...,n$, resulting in $x^{(0)}=X^{(1)}D^{1}$, where D^{1} represents the 1-$IAGO$ matrix, which can be expressed as follows

$$D^{1}=\begin{bmatrix}1 & -1 & 0 & \cdots & 0\\ 0 & 1 & -1 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & 1\end{bmatrix}_{n\times n}. \tag{7}$$

2.2 Fractional FGM (1,1) model establishment

Definition 3. Let $X^{(0)}=\left(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\right)$ be the original sequence, $X^{(r)}=\left(x^{(r)}(1),x^{(r)}(2),\cdots,x^{(r)}(n)\right)=X^{(0)}A^{r}$, $X^{(r)}$ be the rth-order accumulation of $X^{(0)}$ to generate the sequence ($r\text{-}AGO$).

Let $Z_1^{(r)}=\left(z_1^{(r)}(2),z_1^{(r)}(3),\cdots,z_1^{(r)}(n)\right)$ be the sequence of the nearest mean of $X^{(r)}$, where

$$z_1^{(r)}(k)=\frac{1}{2}\left(x^{(r)}(k)+x^{(r)}(k-1)\right), k=2,3,\cdots,n. \tag{8}$$

Let the gray differential equation

$$x^{(r-1)}(k)+a_1 z_1^{(r)}(k)=b_1. \tag{9}$$

be the rth-order cumulative gray FGM (1,1) model. In particular, when $r=1$, $x^{(r-1)}(k)+a_1z_1^{(r)}(k)=b_1$ becomes $x^{(0)}(k)+a_1z_1^{(r)}(k)=b_1$, the mean GM(1,1) model. Where a_1 is the development coefficient and b_1 is the gray action. Model parameters a_1 and b_1 can be solved by least squares. Let $\hat{u}_1=[a_1,b_1]^T$, by least squares method be

$$\hat{u}_1=\left(B_1^TB_1\right)^{-1}B_1^TY_1, \tag{10}$$

where

$$Y_1=\begin{bmatrix} x^{(r-1)}(2) \\ x^{(r-1)}(3) \\ \vdots \\ x^{(r-1)}(n) \end{bmatrix}, B_1=\begin{bmatrix} -z_1^{(r)}(2) & 1 \\ -z_1^{(r)}(3) & 1 \\ \vdots & \vdots \\ -z_1^{(r)}(n) & 1 \end{bmatrix}. \tag{11}$$

Definition 4. Assuming that the model parameters are as defined above, the corresponding whitening differential equation can be obtained from grey differential equation (9)

$$\frac{dx^{(r)}(t)}{dt}+a_1x^{(r)}(t)=b_1. \tag{12}$$

Equation (12) is called the whitening differential equation of the rth-order. Cumulative gray GM(1,1) model. To solve equation (12), let $\hat{x}^{(r)}(1)=x^{(0)}(1)$ get the time response sequence as

$$\hat{x}^{(r)}(k)=\left(x^{(0)}(1)-\frac{b_1}{a_1}\right)e^{-a_1(k-1)}+\frac{b_1}{a_1},k=1,2,\cdots,n. \tag{13}$$

The predicted value of $X^{(0)}$ after restoration is

$$\hat{X}^{(0)}=\hat{X}^{(r)}D^r. \tag{14}$$

Where $\hat{X}^{(0)}=\left(\hat{x}^{(0)}(1),\hat{x}^{(0)}(2),\cdots,\hat{x}^{(0)}(n)\right)$, $\hat{X}^{(r)}=\left(\hat{x}^{(r)}(1),\hat{x}^{(r)}(2),\cdots,\hat{x}^{(r)}(n)\right)$.

3 FGM(1,1,k) Model

The classical GM(1,1) model considers the gray action b as an invariant constant. However, with the development of time and space, the gray action b is not a fixed constant. The literature [33] optimizes the gray action b of the GM(1,1) model to bt, which not only extends the GM(1,1) is applicable, and the accuracy of the model is improved.Therefore, this paper optimizes the gray action b of the FGM(1,1) model to bt, and obtains the following model definition.

Definition 5. Let the gray differential equation

$$x^{(r-1)}(k)+a_2{z_2}^{(r)}(k)=b_2k, \tag{15}$$

be an rth -order non-homogeneous FGM(1,1,k) model. In particular, when $r=1$,

$x^{(r-1)}(k)+a_2 z_2^{(r)}(k)=b_2 k$ becomes $x^{(0)}(k)+a_2 z_2^{(1)}(k)=b_2 k$, that is the NGM(1,1) model. Where a_2 is the development coefficient, and b_2 is the gray action.

Definition 6. Assuming that the model parameters are as defined above, the corresponding whitening differential equation (16) can be obtained from grey differential equation (15)

$$\frac{dx^{(r)}(t)}{dt}+a_2 x^{(r)}(t)=b_2 t, \tag{16}$$

Equation (16) is called the whitening differential equation of the rth-order non-homogeneous FGM(1,1,k) model. The rth-order non-homogeneous FGM(1,1,k) model parameters a_2 and b_2 can be solved by the least squares method. Let $\hat{u}_2=[a_2,b_2]^T$, by least squares method be

$$\hat{u}_2=\left(B_2^T B_2\right)^{-1} B_2^T Y_2, \tag{17}$$

Where

$$Y_2=\begin{bmatrix} x^{(r-1)}(2) \\ x^{(r-1)}(3) \\ \vdots \\ x^{(r-1)}(n) \end{bmatrix},\ B_2=\begin{bmatrix} -z_2^{(r)}(2) & 2 \\ -z_2^{(r)}(3) & 3 \\ \vdots & \vdots \\ -z_2^{(r)}(n) & n \end{bmatrix}. \tag{18}$$

Since equation (15) is a non-homogeneous linear differential equation, the solution is solved by the constant variation method to obtain a general solution

$$x^{(r)}(t)=Ce^{-a_2 t}+\frac{b_2 t}{a_2}-\frac{b_2}{a_2^2}. \tag{19}$$

Let $x^{(r)}(1)=x^{(0)}(1)$, get the time response sequence as

$$\hat{x}^{(r)}(t)=\left(x^{(0)}(1)-\frac{b_2}{a_2}+\frac{b_2}{a_2^2}\right)e^{-a_2(t-1)}+\frac{b_2 t}{a_2}-\frac{b_2}{a_2^2}, t=1,2,\cdots,n. \tag{20}$$

Discretizing equation (20) yields

$$\hat{x}^{(r)}(k)=\left(x^{(0)}(1)-\frac{b_2}{a_2}+\frac{b_2}{a_2^2}\right)e^{-a_2(k-1)}+\frac{b_2 k}{a_2}-\frac{b_2}{a_2^2}, k=1,2,\cdots,n. \tag{21}$$

The predicted value of $X^{(0)}$ after restoration is

$$\hat{X}^{(0)}=\hat{X}^{(r)}D^r. \tag{22}$$

Where $\hat{X}^{(0)}=\left(\hat{x}^{(0)}(1),\hat{x}^{(0)}(2),\cdots,\hat{x}^{(0)}(n)\right)$, $\hat{X}^{(r)}=\left(\hat{x}^{(r)}(1),\hat{x}^{(r)}(2),\cdots,\hat{x}^{(r)}(n)\right)$.

4 SFGM (1,1, k) Model

4.1 Model error analysis

From the time response sequence of FGM(1,1,k), the prediction accuracy of the FGM(1,1,k) model depends on the parameters a_2 and b_2, whereas the parameters a_2, b_2 are mainly determined by the background value $z_2^{(r)}(k)$.For $\frac{dx^{(r)}(t)}{dt}+a_2x^{(r)}(t)=b_2t$ to integrate in $[k-1,k]$, there are

$$I=\int_{k-1}^{k}dx^{(r)}(\mathrm{t})+a_2\int_{k-1}^{k}x^{(r)}(t)dt=b_2\int_{k-1}^{k}tdt, k=2,3,...,n. \tag{23}$$

equation (23) combined with $x^{(r-1)}(k)+a_2z_2^{(r)}(k)=b_2k$, we can get

$$z_2^{(r)}(k)=\int_{k-1}^{k}x^{(r)}(t)dt. \tag{24}$$

The geometric meaning of the background value of the FGM(1,1,k) model are shown in the figures below. It can be clearly seen that the area of the curved trapezoid surrounded by the curve $x^{(r)}(t)$ on the interval $[k-1,k]$ and the horizontal axis t is the background value of the FGM(1,1,k) model. The traditional $z_2^{(r)}(k)$ calculation uses a trapezoidal area to approximate the area of the curved trapezoid, as shown in Fig.1. If the data sequence changes greatly, the background value calculated by the trapezoidal formula will have a large calculation error, resulting in low accuracy of the model and limited application range. However, in the FGM(1,1,k) model improved by Simpson's formula, the background value calculation error is significantly smaller than the error generated by the trapezoidal formula. The background error calculation error of the FGM(1,1,k) model improved by the Simpson formula is shown in Fig. 2.The shaded part in Fig.2 is the calculation error of the Simpson formula relative to the trapezoidal formula.

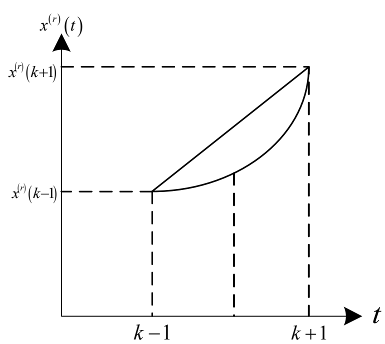

Fig. 1. The trapezoidal formula calculates the background value

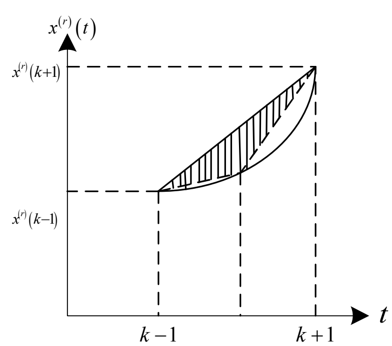

Fig. 2. The Simpson formula calculates the background value

4.2 Improved FGM(1,1,k) model of Simpson formula

Since the FGM(1,1,k) model uses the trapezoidal formula to calculate the background value, this results in a certain error in the model, which makes the prediction accuracy of the FGM(1,1,k) model not high. Therefore, this paper uses the Simpson numerical integration formula to improve the background value calculation of the FGM(1,1,k) model, and obtains the SFGM(1,1,k) model to improve the prediction accuracy of the model.

Consider the integral of (24) in interval $[k-1,k+1]$, we can get

$$I=\int_{k-1}^{k+1}dx^{(r)}(t)+a_3\int_{k-1}^{k+1}x^{(r)}(t)dt=b_3\int_{k-1}^{k+1}tdt,k=2,3,...,n-1. \tag{25}$$

Equation (25) can be further expressed as

$$I=x^{(r)}(k+1)-x^{(r)}(k-1)+a_3\int_{k-1}^{k+1}x^{(r)}(t)dt=2b_3k \quad k=2,3,...,n\text{-}1. \tag{26}$$

Using the Simpson formula to approximate the integral of $\int_{k-1}^{k+1}x^{(r)}(t)dt$ on $[k-1,k+1]$, we can get

$$I_1=\int_{k-1}^{k+1}x^{(r)}(t)dt=\frac{1}{3}\left[x^{(r)}(k-1)+4x^{(r)}(k)+x^{(r)}(k+1)\right] \quad k=2,3,...,n-1. \tag{27}$$

Then equation (26) can be expressed as

$$\begin{aligned}I&=x^{(r)}(k+1)-x^{(r)}(k-1)+a_3\int_{k-1}^{k+1}x^{(r)}(t)dt\\&=x^{(r)}(k+1)-x^{(r)}(k-1)+\frac{a_3}{3}\left[x^{(r)}(k-1)+4x^{(r)}(k)+x^{(r)}(k+1)\right]\\&=2b_3k \quad k=2,3,\cdots,n-1.\end{aligned} \tag{28}$$

The background value $z_3^{(r)}(k)$ obtained by comparing (26) is

$$z_3^{(r)}(k)=\frac{1}{6}\left[x^{(r)}(k-1)+4x^{(r)}(k)+x^{(r)}(k+1)\right] \qquad k=2,3,...,n-1. \tag{29}$$

Further simplification of equation (29), we can get

$$\frac{(x^{(r)}(k+1)-x^{(r)}(k-1))}{2}+\frac{a_3}{6}\left[x^{(r)}(k-1)+4x^{(r)}(k)+x^{(r)}(k+1)\right]=b_3k \quad k=2,3,...,n\text{-}1. \tag{30}$$

Remember that $\hat{u}_3=[a_3,b_3]^T$ is a parameter column, which is known by the principle of least squares. The parameter column needs to satisfy

$$\hat{u}_3=(B_3^TB_3)^{-1}B_3^TY_3. \tag{31}$$

Where

$$Y_3=\begin{pmatrix}\frac{x^{(r)}(3)-x^{(r)}(1)}{2}\\ \frac{x^{(r)}(4)-x^{(r)}(2)}{2}\\ \vdots\\ \frac{x^{(r)}(n)-x^{(r)}(n-2)}{2}\end{pmatrix},\quad B_3=\begin{pmatrix}-\frac{\left(x^{(r)}(1)+4x^{(r)}(2)+x^{(r)}(3)\right)}{6} & 2\\ -\frac{\left(x^{(r)}(2)+4x^{(r)}(3)+x^{(r)}(4)\right)}{6} & 3\\ \vdots & \vdots\\ -\frac{\left(x^{(r)}(n\text{-}2)+4x^{(r)}(n\text{-}1)+x^{(r)}(n)\right)}{6} & n\text{-}1\end{pmatrix}. \tag{32}$$

The time response sequence of the improved SFGM(1,1,k) model of Simpson's formula is

$$\hat{x}^{(r)}(k)=\left(x^{(0)}(1)-\frac{b_3}{a_3}+\frac{b_3}{a_3^2}\right)e^{-a_3(k-1)}+\frac{b_3k}{a_3}-\frac{b_3}{a_3^2},k=1,2,\cdots,n. \tag{33}$$

The predicted value of $X^{(0)}$ after restoration is

$$\hat{X}^{(0)}=\hat{X}^{(r)}D^r. \tag{34}$$

Where $\hat{X}^{(0)}=\left(\hat{x}^{(0)}(1),\hat{x}^{(0)}(2),\cdots,\hat{x}^{(0)}(n)\right)$, $\hat{X}^{(r)}=\left(\hat{x}^{(r)}(1),\hat{x}^{(r)}(2),\cdots,\hat{x}^{(r)}(n)\right)$.

5 Model Test

Model error is an important index to evaluate the reliability and practicability of a model. In this paper, in order to test the prediction accuracy of the model, the mean absolute percentage error ($MAPE_{fit}$) of the fitted data, the mean absolute percentage error ($MPAE_{pred}$) of the extrapolated predicted value and the total mean absolute percentage error ($MAPE_{tol}$) are mainly used to represent the calculation error of the model. The calculation formulas are as follows

$$MAPE_{fit}=\frac{1}{N}\sum_{k=1}^{N}\left|\frac{x^{(0)}(k)-\hat{x}^{(0)}(k)}{x^{(0)}(k)}\right|\times 100\%. \tag{35}$$

$$MAPE_{tol}=\frac{1}{n}\sum_{k=1}^{n}\left|\frac{x^{(0)}(k)-\hat{x}^{(0)}(k)}{x^{(0)}(k)}\right|\times 100\%. \tag{36}$$

$$MPAE_{pred}=\frac{1}{n-N}\sum_{k=N+1}^{n}\left|\frac{x^{(0)}(k)-\hat{x}^{(0)}(k)}{x^{(0)}(k)}\right|\times 100\%. \tag{37}$$

Where n represents the total number of sample data and N represents the number of data used to fit the model. Obviously, the smaller $MAPE_{fit}$, $MPAE_{pred}$ and $MAPE_{tol}$, the better the prediction of the model.

6 Forecast Analysis of Total Import and Export

In order to fully verify the correctness and effectiveness of the proposed SFGM(1,1,k) model, This paper selects and compares the total import and export volume with the similarity of literature and literature [34]. The data is extracted from the total import and export volume of Foreign Economic Relations and Trade in the China Statistical Yearbook from 2000 to 2011. According to the model analysis of the total import and export volume for the past seven years from 2000 to 2006, the total import and export volume for the five years from 2007 to 2011 is predicted. The total import and export data of China from 2000 to 2006 is shown in Table 1.

Table 1. Total imports and exports of foreign trade from 2000 to 2006 (Unit: million USD)

Year	2000	2001	2002	2003	2004	2005	2006
Total import and export	474297	509651	620766	850988	1154554	1421906	1760438

Based on the data in Table 1, the GM(1,1) model, the SGM(1,1) model, the FGM(1,1) model and the SFGM(1,1,k) model are established respectively. And compare and analyze the prediction effect of each model in the total import and export. Taking the $MAPE_{fit}$ as the objective function, the optimal order of FGM(1,1) model and SFGM(1,1,k) model is calculated by MATLAB optimization toolbox as $r1 = 0.6$, $r2 = -0.2$, respectively. The time response equations for the above four models are as follows.

The GM(1,1) predictive model time response equation

$$\hat{x}^{(1)}(k) = 1863562.52e^{0.25(k-1)} \text{-} 1389265.52 \qquad k = 1,2,\cdots,n. \tag{38}$$

The SGM(1,1) predictive model time response equation

$$\hat{x}^{(1)}(k) = 1863562.52e^{0.25(k-1)} \text{-} 332998.32 \qquad k = 1,2,\cdots,n. \tag{39}$$

The FGM(1,1) predictive model time response equation

$$\hat{x}^{(0.6)}(k) = 474305.5e^{0.21(k-1)} \text{-} 8.75 \qquad k = 1,2,\cdots,n. \tag{40}$$

The SFGM(1,1,k) prediction model time response equation

$$\hat{x}^{(-0.2)}(k) = 487762.20e^{-0.94(k-1)} + 217354.43k - 230819.34 \qquad k = 1,2,\cdots,n. \tag{41}$$

According to the time response equation of each model, the total import and export volume from 2000 to 2006 was modeled and fitted, and the total import and export volume from 2007 to 2011 was predicted. Comparing the fitting results with the real data can be obtained as shown in Table 2 below, and the fitting prediction effects of the different models are shown in Fig. 3.

According to Table 2, the fitted mean absolute percentage error $MAPE_{fit}$ of the SFGM(1,1,k) model is 7.7632, while the GM(1,1) model, the Simpson formula improved GM(1,1) model and FGM(1,1) The average absolute percentage error $MAPE_{fit}$ of the model is 33.4655, 38.3363, and 25.9517, respectively. At the same time, according to Fig.1, it can be seen that the prediction accuracy of the SFGM(1,1,k) prediction model is significantly better than other models, which can provide more accurate and reliable prediction information.

Table 2. Comparison of the fitting results of the three prediction models (unit: million USD)

Year	Actual value	GM(1,1)	SGM(1,1)	FGM(1,1)	SFAGM (1,1,k)
2000	474297	474297	474297	474297	474297
2001	509651	520394.4	515614.9	516577.8	488965.4
2002	620766	665713	664184.2	669611	631161
2003	850988	851611.4	855562.3	864651.5	855642.6
2004	1154554	1089421	1102084	1105711	1128262
2005	1421906	1393639	1419638	1403164	1430647
2006	1760438	1782809	1828693	1770743	1753221
$MAPE_{fit}$		**33.4655**	**38.3363**	**25.9517**	**7.7632**

As can be seen from Table 3, the GM(1,1) model, the Simpson modified SGM(1,1) model, and the FGM(1,1) model are respectively 43.1873, 50.2141, 32.9389; 234.2585, 268.3539, and 181.6617, respectively. The $MPAE_{pred}$ and $MAPE_{tol}$ of the SFGM(1,1,k) prediction model are only 8.9520, 54.3422, which are lower than the other three models.

Table 3. Shows the prediction of different models

Year	Actual value	GM(1,1)	SGM(1,1)	FGM(1,1)	SFGM(1,1,k)
2007	2176175	2280653	2355613	2225662	2090822
2008	2563255	2917519	3034359	2789304	2440478
2009	2207535	3732227	3908679	3488209	2800336
2010	2974001	4774441	5034926	4355319	3169140
2011	3641864	6107690	6485689	5431528	3545976
$MPAE_{pred}$		**43.1873**	**50.2141**	**32.9389**	**8.9520**
$MAPE_{tol}$		**234.2585**	**268.3539**	**181.6617**	**54.3422**

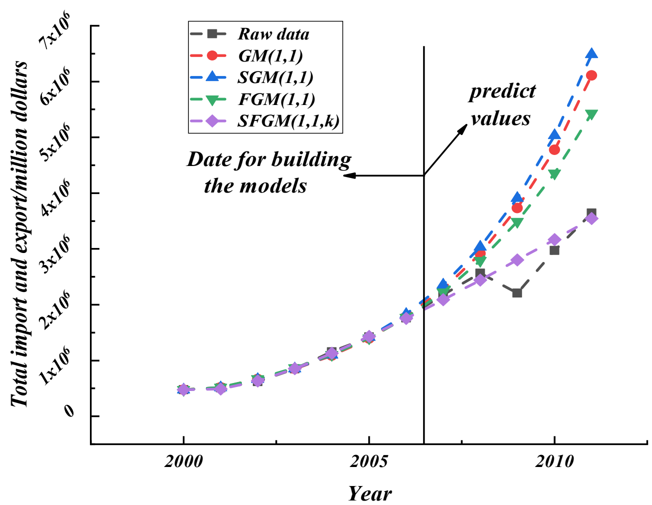

Fig. 3. Fit and prediction effects of different models.

Table 4. Relative errors of the four prediction models

Year	GM(1,1)/%	SGM(1,1)/%	FGM(1,1)/%	SFGM(1,1,k)/%
2000	0.0000	0.0000	0.0000	0.0000
2001	2.1080	1.1702	1.3591	4.0588
2002	7.2406	6.9943	7.8685	1.6745
2003	0.0733	0.5375	1.6056	0.5470
2004	5.6414	4.5446	4.2305	2.2772
2005	1.9880	0.1595	1.3181	0.6147
2006	1.2708	3.8772	0.5854	0.4100
2007	4.8010	8.2456	2.2740	3.9222
2008	13.8209	18.3791	8.8188	4.7899
2009	69.0676	77.0608	58.0138	26.8535
2010	60.5393	69.2981	46.4464	6.5615
2011	67.7078	78.0871	49.1414	2.6329
Mean Relative Error	**19.5215**	**22.3628**	**15.1385**	**4.5285**

Further, in order to more intuitively display the prediction effects of each model, the relative error of the above different prediction models and the MAPE are shown in Fig. 4. It is easy to see that the SFGM(1,1,k) prediction model proposed in this paper improves the fitting accuracy better. At the same time, the prediction accuracy of the model has also been significantly improved. The rationality and effectiveness of the improved method in this paper are verified.

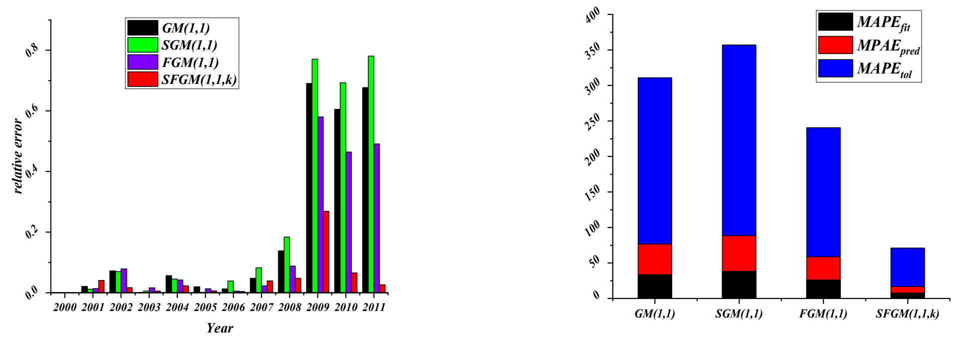

Fig. 4. Error comparison of four prediction models.

7 Conclusion

Based on the classical fractional grey prediction theory, this paper analyzes the error source of the prediction model and reconstructs the background value of the model by Simpson numerical integration formula, which reduces the prediction error. At the same time, the characteristics of the model error distribution and the limitations of a single improved method are considered. Starting from the model parameters, this paper introduces the time term and improves the fitting accuracy of the model by modifying the gray action b to bt. Finally, the SFGM(1,1,k) prediction model proposed in this paper is applied to the total import and export case data. The results show that the SFGM(1,1,k) model can provide more accurate and reliable prediction information, and verify the practicability and effectiveness of the SFGM(1,1,k) model. At the same time, it also provides a theoretical guidance background for the development of foreign economic and trade.

Competing Interests

Authors have declared that no competing interests exist.

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