

Welfare-Enhancing Accumulation of Foreign Reserves*

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ABSTRACT

This paper considers if huge accumulation of foreign reserves by some countries is optimal in a simple, intertemporal, and welfare maximization model with loss aversion. The optimality condition is shown to depend on several underlying parameters of the model. Configuration of output shocks and probability of bad state reveal that, controlling other parameters, huge accumulation of foreign reserves of China and Japan is consistently interpreted as optimal within the model. We also consider if external debts serve as alternative optimal precautionary methods. The optimal precautionary saving is also shown to be welfare-enhancing with loss aversion.

Keywords: Foreign Reserves; Loss Aversion; Precautionary Saving

1. Introduction

It has been recognized that foreign reserve accumulation is costly for governments, and the costs must be compensated by the benefits from reserve holding. In the real world, foreign reserves have been accumulated, but the essential problem has not only been the accumulation itself, but also its uneven distribution across developed and developing nations. This uneven holding is a source of the unresolved issue called as the global imbalance (Corden [1], McKinnon [2] and Roubini [3]). For example, at the end of 2010, reserve of China and Japan constituted 2300 billion SDRs, more than 40 percent of the total world foreign reserves (minus gold). From historical data, foreign reserves of China and Japan have increased drastically by 11.4 and 2.4 times over the last decade (see **Figures 1(a)** and **(b)**)¹.

If reserve holding is costly, it is also puzzling that such huge reserves have been accumulated by some countries, because the demand for reserves was believed to diminish since the advent of the general floating from 1973. Contrary to this prediction, it is a historically observed fact that accumulation grew fast under exchange rate flexibility, known as the Frenkel's paradox in the literature

(Bastourre *et al.* [4] and Frenkel [5])².

The optimal level of foreign reserves is defined as the level that maximizes the (utility of) net benefit which is, in turn, defined as the difference between the total benefit and the total cost associated with reserve holding. However, the optimal level has long been examined mainly from the *cost* side, while keeping the benefit side fixed consciously or unconsciously. There are several excellent surveys focusing on the minimization of the costs (Bahmani-Oskooee *et al.* [6] and Flood *et al.* [7]). It has been a relatively new tendency to consider the optimal level from the *benefit* side. In the next Section 2, the author will present a non-exclusive survey of the recent literature of the optimal reserves examined mainly from the benefit side.

Although several rules of thumb have been proposed for practical level of foreign reserves for policy makers such as maintaining reserves equivalent to three months of imports or full coverage of total short-term foreign debt known as the "Greenspan-Guidotti" rule (Jeanne and Rancière [8,9]; Obstfeld *et al.* [10])³, recent literature based on rigorous optimization frameworks propose that foreign reserves are explained by such motivations as self-insurance (or precautionary motive or buffer stock; e.g., Aizenman *et al.* [11-13], Cifarelli and Paladino [14]), financial

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¹The source is the IMF-IFS On-line. The Figure makes it clear that foreign reserves have been unevenly distributed, and the vector is now moving clearly from advanced countries to emerging and developing countries during the last decade (**Figure 1(b)**).

²According to Frenkel [5], central banks' behavior did not change substantially before and after the Bretton-Woods breakdown in 1973. Before the breakdown, many of the exchange rate arrangements were adjustable peg, and they were managed (not pure) floating afterwards.

³Huge foreign reserves of China and Japan cover their imports for 8.3 and 6.5 quarters, respectively, as of December 2010. Also, their foreign reserves are more than 10.2 and 7.9 times larger than those suggested by the "Greenspan-Guidotti" rule, as of December 2010. For the rules of thumb results for other Asian countries, see, e.g. Park and Estrada [18].

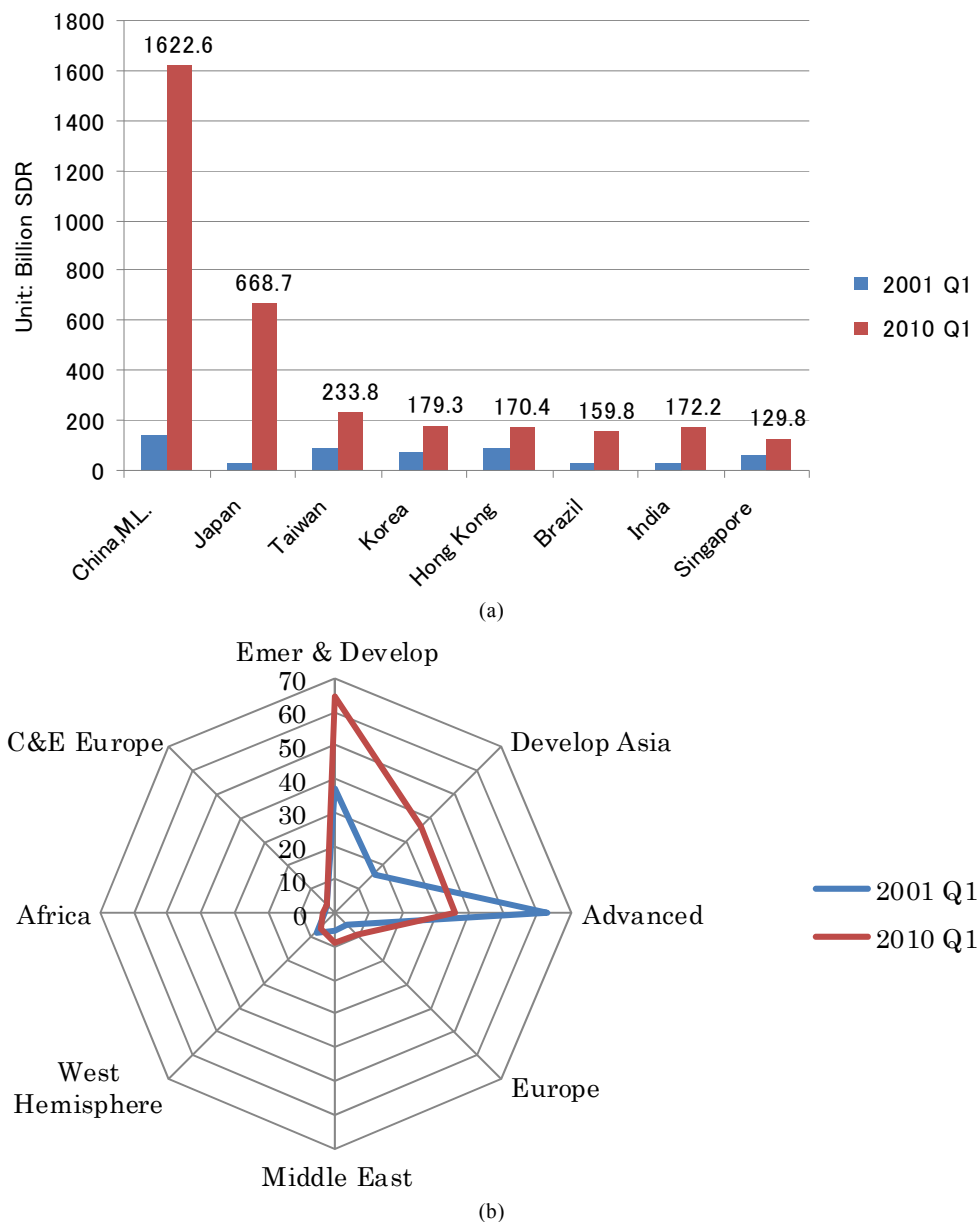


Figure 1. (a) Large holders of foreign reserves. Data source: IMF-IFS on-line, accessed on April 28, 2011; (b) Regional distribution of foreign reserves. Data source: see (a).

stability “to support the overall banking system while avoiding extreme currency depreciation” (Obstfeld *et al.* [10]: p. 11) or financial mercantilism proposed by Aizenman and Lee [15,16]. For example, the second motivation, financial stability, suggests that the optimal level of reserves set by emerging market policymakers reflects a risk of “double drain”, internal drain of deposits and external drain of reserves, and thus the “demand for international reserves may go far beyond what would be needed simply to insure a “sudden stop” in foreign capital inflows” (p. 11). Likewise, Chang and Velasco [17] argue that it is not socially optimal for governments to hold a precautionary *war chest* of foreign reserves for

fear of balance of payments crises with fixed exchange rates, simply because the social marginal rate of substitution is not equated to the social marginal rate of transformation⁴. Furthermore, the financial mercantilism argument is based on a game-theoretic model to explain the hazard of competitive accumulation. According to Aizenman and Lee [15], it is possible to find a Nash-equilibrium solution with high levels of reserves by export-led growth policies, as a result of “utility” maximization (p. 9). Some countries adopt exchange rate policies to keep their

⁴According to them, the reason for this non-optimality stems from the implicit cost of building the war chest, because the government keeps all of the war chest in a liquid form instead of investing it optimally.

exchange rate undervalued for a competitiveness purpose. Thus, governments of those countries may prefer to accumulate reserves and maintain some type of capital controls.

Several interesting characteristics of the optimal level of foreign reserves have been clarified by those recent researches, but there still remain unresolved or unclear issues related to reserves. There are several purposes of this paper. 1) First is to present a “pure” insurance model of foreign reserve, in the sense it neither incorporate financial stability reflecting double drain nor financial mercantilism; and then 2) Second is to construct a simple intertemporal model of foreign reserves with loss aversion⁵. A special emphasis is given to the probability of bad state, e.g. crisis, and we consider how the national welfare level changes depending on the probability and loss aversion; 3) Third, we offer our new, consistent, interpretation of Chinese and Japanese huge accumulation within an intertemporal welfare-maximizing model with loss aversion. We also examine the welfare when a country borrows for fear of a bad state from abroad, and consider the substitutability for reserves; 4) Fourth, completing the present investigation, we then consider whether precautionary saving within our simple model with loss aversion is warranted.

The paper is organized as follows. Section 2 presents a brief survey of the literature which emphasizes the *benefit* side of welfare. Section 3 formulates a “pure” precautionary and insurance model followed by a simple dynamic model with loss aversion in Section 4. Section 5 examines whether national welfare is improved by precautionary holding of foreign reserves. The effects of changes in exogenous parameters are examined in Section 6. Section 7 is devoted to examine the possibility of international debts as an alternative precautionary device. The optimal saving in a simple intertemporal utility maximization model with loss aversion is examined in Section 8. Section 9 concludes the paper.

2. Literature Reviewed⁶

Since the preceding literature on the problems related to the optimal foreign reserves have been reviewed and discussed in excellent survey articles by Bahmani-Oskooee and Brown [6] and Flood and Marion [7], our review here is confined as concise as possible. The former survey even mentioned that there were at least seven earlier review articles prior to it. Thus, our task of this re-

view should focus primarily on new points of view addressed since late 1990s.

It seems to the author that several biases have been observed in the articles reviewed in those preceding surveys. The first bias is that, although the theories of the demand for the foreign reserves have been extensively emphasized and examined, discussions on the supply side have been relatively scarce (Bahmani-Oskooee *et al.* [6]). Secondly, although empirical investigation has grown rapidly after the generalized floating exchange rate regime since 1973, it has been totally unclear that why hypothetical statistical models can be used to estimate an “appropriate” level or the “optimal” demand for the foreign reserves. It seems that this unfounded bias is blamed for an explicit or implicit assumption of regarding the actual level as the “optimal” or “appropriate” level of the foreign reserves. The third bias is related to the second one, disregarding close scrutiny of the “optimality”. The economic significance of “optimality” absolutely dictates that the models either maximize some welfare benefits (*i.e.* utility or profits) or minimize some costs, or both. The fourth bias is found in empirical investigation because of theoretical examination on the optimality of welfare benefits has been premature. As a result, the most of statistical investigation has been dominated exclusively by models of *cost minimization* put forth by e.g. Heller [21] or Frenkel and Jovanovic [22]. However, it is surprising that no single literature has been found that examines and approaches to welfare implications by utility or profit *maximization*, except those to be reviewed below.

Many countries that observed or experienced financial crises during 1980s and 1990s have since been accumulating foreign reserves under the uncertainty of exchange rates, with their perception of non-robustness of the so-called “soft-peg” regimes⁷. Several motives for holding foreign reserves are pointed out. Bahmani-Oskooee and Brown [6] (p. 1210) summarize five distinct reasons: to smooth out temporary payments imbalances, to defuse a speculative run on currencies, to build prestige, to impawn for collateral, and to intervene foreign exchange market. However, as the previous literature has been biased as explained above, here we focus on the third bias of blur treatment of optimality. In view of an unreasonable emphasize on the cost minimization in the preceding literature as pointed out as the fourth bias, a novel feature of our review below is that here we intentionally review some important recent *theoretical* contributions on the “optimal” level of foreign reserves that explicitly consider the welfare gains from the *maximization of benefits*⁸. Thus, we

⁵“Loss aversion” is defined as the tendency of agents to be more sensitive to reductions in their consumption plan than to increases relative to a reference point (Aizenman [19]; Aizenman *et al.* [12,13]).

⁶In addition to several rules of thumb for practical level of foreign reserves mentioned earlier, there has been another strand of literature to discuss the dynamics of reserve accumulation, known as the “Mrs. Machlup’s Wardrobe” hypothesis. However, we disregard the hypothesis here, since it has not been well-based on the *optimality*. For a recent empirical study on the hypothesis, see Cheung and Sengupta [20].

⁷Although foreign reserves were expected to diminish after the onset of a generalized floating regime since 1973, it is puzzling that the accumulation began to grow fast. See Bastourre, Carrera and Ibarlucia [4]. One of the possible reasons is the “Fear of Floating” (Akiba *et al.* [23]).

⁸In this respect, our review is necessarily biased since the cost side of foreign reserves is either disregarded or implicitly kept constant.

totally disregard empirical articles in our discussion below simply because they have been exclusively examined through the cost side of foreign reserves. One additional new point of view is examination of effects of international debts on the level of foreign reserves, as the monetary authority (or government agencies) might determine both of them jointly through the budget constraint.

Moreover, Aizenman [19,12] first formulates a model for the optimal level of foreign reserves by applying a microeconomic utility maximization problem. Using the concept of “disappointment aversion” by Gull [24], he derives the government’s optimal level of foreign reserves by applying an individual’s expected utility function that exhibits “disappointment” from a smaller consumption than the certainty equivalent level under income uncertainty. The “disappointment aversion” measure has been called “loss aversion” after expanding its domain over total wealth. Applying the concept of “loss aversion” within a framework of buffer stock and foreign reserves, Aizenman [19,12] shows that the foreign reserves as buffer stock in fact depend on disappointment aversion (or loss aversion) in a tractable way. Specifically, it rigorously shows that the optimal foreign reserves are increasing with loss aversion, and the optimal level is likely to be positive under income uncertainty.

In order to show the importance of loss aversion, Aizenman and Marion [11], recognizing that the Korean financial crisis since 1997 was caused by a fall in willingness of international loans due to a decrease in the Korean reserve level, show that a change in reserves has an asymmetric effect, especially when an expected large fall in reserves causes a large reduction in the supply of international credit when the private sector downgrades its prior belief towards repayment possibilities or becomes more pessimistic about the future level of reserve position. This asymmetric effect is the essence of the notion of disappointment (or loss) aversion.

Both Aizenman and Marion [12,13] and Alfaro and Kanczuk [25] consider an intertemporal, 2-period, and dynamic stochastic model under income uncertainty. They examine whether a country, maximizing the present discounted value of the expected utility, holds foreign reserves for a precautionary purpose. The novel feature of the model lies in their assumption that the government maximizes the country’s welfare by jointly choosing the optimal levels of foreign reserves *and* international debts. In the former article, the government finances its foreign reserves’ costs by collecting taxes. Moreover, while the former article shows that the optimal foreign reserve level is unambiguously positive when the country’s historical reserve position is zero, the latter article also calibrates the optimal dynamic course of foreign reserves, showing that the optimal level is zero. The differences reveal two interesting facts toward the model. First the positive re-

serve level in the former article is shown only at the point of time where the initial holding is zero, while the latter article examines the future reserve level is zero. Second, while the former article has not considered the possibility of default, the latter article assumes it as a possible choice. The common feature of both studies is that the optimal reserves decrease as the discount factor decreases. This is a plausible result, as it implies more consumption in an earlier period and tilts the tax rates toward a later period. Thus, international reserve holding must fall, while external borrowing must rise to satisfy the budget constraint.⁹ But, the latter article shows that both are not perfect substitutes because of possibility of default. Since the most of the latter article’s results are based only on calibration with specific assumptions of an isoelastic utility function and GDP being fitted by an AR(1) process, and even though their several robustness exercises, their conclusion of the optimal level of foreign reserves being zero remains to be explained in view of the large actual accumulation of foreign reserves in many developed, developing, and emerging economies.

Aizenman and Lee [15,16] consider whether foreign reserves for a precautionary purpose is consistent with a mercantilist interpretation. They distinguished two different mercantilisms: financial and monetary. The former one is characterized by direct subsidies, financial repressions, or moral suasion, while the latter one hinges on hoarding foreign reserves. As long as both mercantilisms have a negative beggar-thy-neighbor externality associated with costs, large reserve holding is inefficient. While their article [15] deduces that monetary mercantilism is observationally near-equivalent to precautionary holding, they come up from their regression in their article [16] with a conclusion that the precautionary motive plays a more visible role than the mercantilist motive represented by the growth rate of real exports and national price levels. Then, they construct a two-period dynamic model of a private bank embodying a precautionary motive by self-insurance. The model is similar in spirit to Aizenman and Marion [12,13] that the bank borrows (accepts deposits) and either invests in real capital or hoards as reserves. Thus, the model is a straightforward application of the familiar bank run model by Diamond and Dybvig [26]. By maximizing the expected profits, they derive that the optimal reserves are held up to the point where the expected opportunity cost of hoarding reserves equals the expected precautionary benefit. Then, they calculate the optimal demand for deposit and the profit, and they simulate it under a liquidity shock defined by fluctuations in the ratio of reserves to deposit. They show a plausible result that the reserve ratio increases with the volatility.

⁹According to Aizenman and Marion ([12]: p. 394), the effect of the decrease in the discount factor is “very similar to the effect of political uncertainty or corruption.”

In addition, they also show that the deposit is increasing with reserves, because an increase in reserves means a decrease in investment, and hence output. Thus, the costs associated with this loss in output must be mitigated by an increase in deposit.

Jeanne and Ranci ere [8,9] construct an intertemporal aggregate model with uncertainty in a sudden stop where the government maximizes the present discounted expected utility of a representative individual. The period utility or felicity function is assumed to be isoelastic to derive closed-form solutions. Uncertainty in sudden-stop is shown to justify holding foreign reserves as self-insurance for a precautionary purpose. When the capital account runs a deficit by a sudden stop, the deficit implies a surplus in the current account because of the balance of payments identity, causing a fall in absorption. Upon optimization, the optimal level of foreign reserves in normal time (*i.e.* no sudden-stop) is shown as a fixed fraction of the output level. This fraction is approximately equal to the sum of the short-term debts to GDP and the output cost of a sudden stop.

A general conclusion emerging from our brief literature review so far of the theoretical elaboration is that, although the optimal level of foreign reserves is positive when considered independently from external debts, this conclusion may be somehow modified once they are considered jointly for optimization. Since our preceding literature review focuses on the “optimality” of foreign reserves from a different standpoint of (expected) utility or profit maximization, the purpose of this study continues to examine the welfare (*i.e.* the social utility) implications by emphasizing precautionary holding of reserves within a framework of loss aversion theory, and synthesizes the latter framework with a simple dynamic two-period model under income uncertainty with joint determination of foreign reserves and international debts. We also consider a defaulting case for external debts as a possible alternative choice when the state of nature turns from “good” to “bad”.

3. Precautionary Foreign Reserve as Simple Insurance

This section considers a “pure” insurance model of foreign reserve, in the sense it neither incorporate financial stability reflecting double drain nor financial mercantile-ism. This model may be interpreted as reflecting the essential characteristics of the optimal, precautionary, foreign reserves in a static model. Suppose a country faces a possible output loss of $x (>0)$ with probability p at a given period. For simplicity, we assume only two states of nature with loss and no loss. Utility and output are denoted by U and Y , respectively. Thus, the expected U , U_0 , is given by:

$$U_0 = (1 - p)U(Y) + pU(Y - \alpha) \tag{1}$$

Since utility is decreased by $\alpha (\alpha \leq Y)$, the country may decide to hold precautionary saving, denoted by h . This idea is consistent with Bastourre, Carrera, and Ibarlucia [4] who consider that precautionary holding of saving for foreign reserves works as *insurance*. We simply assume that h covers full amount in case of loss of $\alpha (h = \alpha)$. The expected utility in this case, U_1 , is:

$$U_1 = (1 - p)U(Y - h) + pU(Y - h) \tag{2}$$

For the precautionary saving h to be meaningful, $U_1 > U_0$ must hold. Since $dU_1/dh = -U'(Y - h) < 0$, the maximum amount that this country saves, h^* , must satisfies:

$$U(Y - h^*) = (1 - p)U(Y) + pU(Y - \alpha) \tag{3}$$

It immediately follows from (3) that:

$$U(Y - \alpha) < U(Y - h^*) < U(Y) \tag{4}$$

Thus, because of the monotonicity of U , $U' > 0$, it follows that:

$$h^* < \alpha \tag{5}$$

Thus, the optimal foreign reserves must be smaller than the possible output loss.

Assuming the concavity of utility function, differentiation of Equation (3) for a given Y yields:

$$dh^*/dp = -[U(Y - \alpha) - U(Y)]/U'(Y - h^*) > 0 \tag{6}$$

Thus, the optimal level of foreign reserves as buffer stock is increased with an increase in the probability of loss.

Because of concavity of U , *Jensen's inequality*, $U(E[Y]) > E(U[Y])$, holds where E is the expectations operator. Thus,

$$U[p(Y - \alpha) + (1 - p)Y] > pU(Y - \alpha) + (1 - p)UY \tag{7}$$

It then follows from (3) and (7) that:

$$U(Y - p\alpha) > U(Y - h^*) \tag{8}$$

The monotonicity assumption of U implies that:

$$h^* > p\alpha (= E[\alpha]) \tag{9}$$

Thus, inequalities (5) and (9) imply that the optimal level of foreign reserves as buffer stock is larger than the expected loss ($E[\alpha]$), but smaller than the possible output loss (α). Since $p\alpha (= E[\alpha])$ increases with an increase in p , h^* also increases, implying that the minimum level of buffer stock increases with p . It can easily be shown that this conclusion is not altered when h is assumed to cover only a part of loss ($h < \alpha$). In the following sections, we will take a closer look at the expected utility and the effects of loss probability, p , as policymakers may not conceive a good and a bad state symmetrically even when p is a half (Tversky and Kahneman [27]).

4. Precautionary Reserve Holding in a Simple Dynamic Model with Loss Aversion

This section constructs a buffer stock model to derive the demand for international reserves under loss aversion. We presuppose that policymakers are not only interested in financial stability or financial mercantilism, but also their primary concern is to avert additional “disutility” arising from a bad state which differs from utility of a good state under uncertainty (Aizenman [19]). The model is formulated in a simple but standard intertemporal model (e.g. Obstfeld and Rogoff (OR, hereafter), [28], Chapter 1). It is assumed that the policy authority of a country maximizes the social welfare summarized in a well-behaved period utility (or felicity) function, and the authority may hold foreign reserves as precautionary savings in the presence of loss aversion¹⁰. It should be emphasized that the government’s maximization is determined *after* recognizing the private sector’s maximization.

The private sector is assumed to maximize the discounted present value of their utility function for two periods,

$$U_1 = u(C_1) + \beta E[u(C_{2s})] \quad (10)$$

subject to the discounted present value of their budget constraint of the form of

$$C_1 + I_1 + (C_2 + I_2)/(1+r) = Y_1 + Y_2/(1+r) \quad (11)$$

where u is the utility function, C_1 is consumption in the first period, C_{2s} is that in the second period when state s is realized, E is the expectation operator, I_t ($t = 1, 2$) is the investment, Y_t ($t = 1, 2$) is the output in the t -th period, β is the discount factor, and r is the real rate of interest determined in the world capital market and thus exogenously taken by this small country. The production function for new output in each period is given by

$$Y_t = F(K_t)(1+\gamma) \quad (12)$$

where K stands for capital. As usual, it is assumed that $F(0) = 0$ and $F'(0) = \infty$, and that the marginal product of capital (MPK) is strictly positive, but diminishing with capital input, $F' > 0$ and $F'' < 0$. The production process is assumed reversible, implying that in the planning horizon of two, capital is assumed consumable, with $K_0 = K_3 = 0$. K_1 is assumed given exogenously by historical data. It should be emphasized that the private sector maximizes their lifetime utility (10) in advance *without knowing government’s loss aversion policy by reserves or debts accumulation*.

¹⁰Loss aversion has been widely known in such fields as Marketing or Behavioral Economics as an explanation of asymmetric responses of consumers to a price change. For example, see Han *et al.* [29] for a practical application.

Recognizing the optimal choices by the private sector, it is assumed that the policy authority foresees some risk of exogenous shocks that is reflected in fluctuations of the future (*i.e.* second period) output, Y_2 . Specifically, it is assumed in (12) that, while $\gamma = 0$ in $t = 1$ (*i.e.* no uncertainty), γ is either $+\alpha$ or $-\alpha$ with probability $1-p$ and p , respectively, in period 2 ($\alpha > 0$). Thus, p is the probability of bad state, while $1-p$ is that of good state. Facing with the production uncertainty, it is assumed that the policy authority undertakes two changes in policy (Aizenman [19]; Aizenman and Marion [12]; Jeanne and Rancière [8,9]):

1) In order to compensate for possible output losses in future, the policy authority holds buffer stock as precautionary savings. Thus, they withholds R ($< Y_1$) of resources from the private sector in period 1 that will be repaid in period 2 with interest payments, $(1+r_f)R$. r_f is the safe interest rate which is possibly zero. This real resources R is called “Foreign Reserves”. For simplicity, it is assumed that the management cost of holding R is zero. In addition to R , the policy authority may decide external borrowing from abroad, B , in period 1, which is repaid with the interest in period 2, r_B . They may decide to default without any penalty in period 2, but this option would be costly in the longer-run, as this country will be excluded from the international capital market. It is also assumed that the management cost of holding B is zero, and $r_B \geq r \geq r_f \geq 0$ ¹¹.

2) The motivation of the policy authority for holding buffer stock R or external debts B is assumed to be a “loss aversion”, implying that “losses loom larger than corresponding gains”(Tversky and Kahneman [27]: p. 1047). Thus, at the point of autarky (*i.e.* before international trade), they have an expected utility of the following form in period 2 (see Gul [24]; Aizenman [19]: p. 936; Aizenman and Marion, [12]: p. 395):

$$E[u(C_2)] = (1-p)(1-\lambda_H)u(Y_{2,H}) + p(1-\lambda_L)u(Y_{2,L})$$

As clarified in the Appendix section, λ_H is defined as $p\sigma/(1-p\sigma)$, whereas λ_L is defined as $(1-p)\sigma/(1+p\sigma)$. σ is a parameter which measures loss aversion of the policy authority of this country.

$1 + \lambda_L$ in the expected utility is the extra weight attached to the bad state of the nature where the government would be disappointed (relative to the probability weight, p , used in the conventional utility). Similarly, $1 - \lambda_H$ attached to the good state of the nature means that the government attaches to a lighter weight. By construction, it is clear that λ_s and p are interrelated to each other (see the Appendix section), as in Aizenman [19] and

¹¹For simplicity, it is assumed that the price of debts is unchanged even after default. For a case of changing price, see Alfaro and Kanczuk [25].

Aizenman and Marion [12]¹². Thus, replacing $E[u(C_{2s})]$ by $W(\sigma; \{C_{2s}\})$, the maximization problem for the policy authority under loss aversion is formally presented as:

$$\begin{aligned} \max U_1 &= u(C_1) + \beta W(\sigma; \{C_{2s}\}) \\ &= u(C_1) + \beta \left[(1-p)(1-\lambda_H)u(Y_{2,H}) \right. \\ &\quad \left. + p(1+\lambda_L)u(Y_{2,L}) \right] \end{aligned} \quad (10')$$

σ is called the “loss aversion rate”, and $Y_{2,H}$ is the level of higher output when $\gamma = +\alpha$ (good state), but $Y_{2,L}$ is the lower output level when $\gamma = -\alpha$ (bad state)¹³. The budget constraint is:

$$\begin{aligned} C_1 + I_1 + (C_2 + I_2)/(1+r) \\ = Y_1 + B - R + \left[Y_2 - (1+r_B)B + (1+r_f)R \right] / (1+r) \end{aligned} \quad (11')$$

with production function with capital stock constraints:

$$Y_t = F(K_t)(1+\gamma), K_t = K_{t-1} + I_t, K_0 = K_3 = 0 \quad (12')$$

From the model summarized above, there are three facts worth further emphasizing. First, it is straightforward to confirm that, upon maximization of (10) or (10') at the first stage with respect to consumption and investment, the standard (expected) intertemporal Euler equation and the equality of the MPK and the real rate of interest are established (see, e.g. OR [28], Chapter 1, Section 1.2.2). Recall that we have assumed that the private sector chooses the optimal levels of consumption and investment *before* the monetary authority implements precautionary policy against possible loss in the second period. Secondly, external debts B and buffer stock R are possible substitutes, as they must satisfy the intertemporal budget constraint (11'). Thirdly, we explicitly assume in (11') that the government (or monetary authority) saves R as buffer stocks in the first period, and invests it in safe assets whose interest rate r_f is smaller than r (the marginal product of capital)¹⁴. Thus, for $B = 0$, the present discounted lifetime budget constraint (11') with buffer stock R lies strictly inside of the constraint (11') without it, provided that $r > r_f$, as also confirmed easily by depicting an intertemporal production possibilities frontier (PPF) (see OR [28], Chapter 1, Section 1.2.3). Similarly, for $R = 0$, the budget constraint with external debts B lies strictly outside of the constraint (11') without it if r_B is strictly larger than r , $r_B > r$. Thus, considering from the

¹²For more details of loss aversion, see, e.g. Aizenman [19], Benartzi and Thaler [30], Kahneman, Knetsch, and Thaler [31], where it is postulated that λ_s is also a non-linear function of p , although the functional form is slightly different.

¹³It is assumed here that $E[u(C_2)]$ can be expressed in this way even if the probabilities of two states of the world are different for simplicity. Note that Aizenman [19] consider a special case where λ_H and λ_L are constant, and fixed at $\lambda_H = \lambda_L = 1/2$ (see the Appendix section for more detail). We relax these restrictions in what follows.

¹⁴In what follows, r_f is assumed to be zero for simplicity.

supply side of the economy, the national welfare level is either decreased or increased.

The last observation is directly relevant to our present investigation. Our purpose of this paper, therefore, can be focusing on examining a possibility that precautionary holding of foreign reserves as buffer stocks is welfare improving in the presence of loss aversion, even if the PPF is shrunk inwards. Put differently, we would like to know if it is optimal for the policy authority to hold foreign reserves as buffer stocks under production risk in the presence of disappointment aversion¹⁵. We also address if external debts B affect the optimal level of R .

5. The Optimal Reserve Holding in a Dynamic Model with Loss Aversion

Using the model specified in the last section, this section considers whether there exists the marginal benefit of holding reserves as buffer stocks that reflect loss aversion for the monetary authority. We assume, for simplicity, the rate of interest on the safe assets, r_f , is set to zero. Assume also that the authority neither has foreign reserves nor external debts at the beginning of the first period, or alternatively the historical levels of foreign reserve accumulation and external debts are zero.

The private sector of this economy is assumed to choose their optimal levels of consumption and investment, *without realizing that the monetary authority chooses R optimally as buffer stocks*. This implies that, for the optimal investment decision, I_1 , the following condition is satisfied:

$$\begin{aligned} -u'(Y_1) \\ + \beta \left\{ (1-p)(1-\lambda_H) \left[(1+\alpha)F'(K_2) - r \right] u'(Y_{2,H}) \right. \\ \left. + p(1+\lambda_L) \left[(1-\alpha)F'(K_2) - r \right] u'(Y_{2,L}) \right\} = 0 \end{aligned} \quad (13)$$

Since $K_0 = 0$ is given historically by assumption, and thus $K_0 + I_1 = K_1$, $u'(Y_1)$ measures the MPK in utility unit at the first period under our assumption of autarky. Recall that we have assumed the equality between MPK and the real rate of interest *before* the loss aversion policy implemented by the monetary authority.

After observing (13), the authority considers the marginal benefit accruing from reserve holding¹⁶:

$$\begin{aligned} \frac{\partial U_1}{\partial R} = -u'(Y_1) \\ - \beta r \left\{ (1-p)(1-\lambda_H)u'(Y_{2,H}) + p(1+\lambda_L)u'(Y_{2,L}) \right\} \end{aligned} \quad (14)$$

where Equation (14) is also evaluated at the autarky point (*i.e.* no international trade). Solving the private sector's

¹⁵Note that PPF shrinks inward only when $r_f < r$ (=MPK).

¹⁶More generally it is assumed that the country has neither positive nor negative trade balances carried over from the previous periods at the beginning of the planning horizon.

optimality condition (13) for $u'(Y_1)$ and substituting it in the government's optimality condition (14) yields:

$$\frac{\partial U_1}{\partial R} = \beta \{ -(1-p)(1-\lambda_H)(1+\alpha)F'(K_2)u'(Y_{2H}) - p(1+\lambda_L)(1-\alpha)F'(K_2)u'(Y_{2L}) \} \quad (15)$$

Computing the first-order-approximation of $u'(C_2)$ around the neighborhood of $\alpha = 0$, making use of the definition of the (Arrow-Pratt) measure of relative risk aversion $(-Yu''/u')$, and denoting the latter by ϕ , we end up with $u'(C_{2H}) = u'(C_2)(1-\alpha\phi)$ and $u'(C_{2L}) = u'(C_2)(1+\alpha\phi)$. Upon substitution of these approximated expressions into (15) yields:

$$\frac{\partial U_1}{\partial R} = \beta u'(Y_2)F'(K_2) \times \{ -(1-p)(1-\lambda_H)(1+\alpha)(1-\alpha\phi) - p(1+\lambda_L)(1-\alpha)(1+\alpha\phi) \} \quad (16)$$

As explained earlier, loss aversion implies that λ_H and λ_L are respectively given by non-linear functions of p (see Appendix):

$$\lambda_H \equiv \frac{p\sigma}{1+p\sigma} \text{ and } \lambda_L \equiv \frac{(1-p)\sigma}{1+p\sigma} \quad (17)$$

Substituting (17) into (16) yields:

$$\frac{\partial U_1}{\partial R} = \frac{\beta u'(Y_2)F'(K_2)}{1+p\sigma} \{ -(1+\alpha)(1-\alpha\phi)(1-p) - (1-\alpha)(1+\alpha\phi)p(1+\sigma) \} \quad (18)$$

which is the desired expression. From (18) it is apparent that, when $\alpha = 0$ (*i.e.*, no uncertainty in the future output), the right-hand side of (18) reduces to $-\beta u'(Y_2)F'(K_2) < 0$, implying that a country has no incentive to hold foreign reserves. However, as also apparent from (18), the sign is likely to be positive, depending on the configuration of the underlying parameters, α , ϕ , σ , and p , implying that the optimal level of demand for foreign reserves is positive at the zero level of foreign reserves at the beginning of the planning period. Thus, a country is *optimal* to hold a positive level of foreign reserves as buffer stocks (*i.e.*, by a precautionary motive due to loss aversion) in order to smooth consumption.

A simple numerical example may help understanding

the possibility of positive demand for foreign reserves due to a precautionary motive. Empirical estimates of the loss aversion ratio, $(1+\lambda_L)/(1-\lambda_H)$, are typically around the neighborhood of 2, implying that $(1+\sigma) = 2$ (Aizenman [19]; Aizenman and Marion 12). Thus, we assume that $\sigma = 1$. The coefficient of relative risk aversion is considered to be in the neighborhood of 2 (the ‘‘Samuelson’s presumption’’). We thus assume that ϕ is 2. Then, at the initial autarky position, if both good and bad state are equally likely, (*i.e.* $p = 0.5$), the bracket term on the right-hand side of (18) is unambiguously **positive** for approximately $0.795 < \alpha < 1$, meaning that a country’s welfare improves by holding foreign reserves as buffer stocks¹⁷.

The probability of bad state, p , is difficult to assess. According to historical data, the probability of crisis is not high. For example, according to Kraay [32], the proportion of currency crises caused by successful speculative attacks against the total attacks in a sample of 54 industrial and developing countries over the period January 1975 to April 1999 is **0.39**¹⁸. According to a more sophisticated method of out-of-sample predictions using 21 ‘‘indicators’’, using a ‘‘signaling’’ approach of an ‘‘Early Warning (EW, hereafter)’’ system Edison [33] also reports crisis probabilities for a sample of 21 developing countries as of December 1996. For example, the simple average probability for 10 Asian countries is **0.28**¹⁹. The reason for this low probability, also commonly found in the literature, is that ‘‘frequently indicators are signaling and no currency follows’’ (Edison [33]: p. 36). Thus, Edison [33] implies that the probability found in Kraay [32] is somewhat overestimated, as some indicators exceed the threshold values of crisis without inducing speculative attacks. In a recent article, Budsayaplakorn *et al.* [34] also replicate the ‘‘indicators’’ method employed by Edison [33] and find that the average probability of crisis for five Asian crisis-hit countries is **0.32** (their Table 5, p. 17). But, according to the estimated probability by a probit model, they report that the average probability is even lower for those countries, **0.19** (their Table 5). A similar assessment was also obtained from an EW model by Ito and Orri [35] who first estimated the crisis model for five Asian countries hit by the 1997 crisis and then predicted out-of-sample forecasts of the crisis probabilities. The average predicted probability for the five countries is **0.278**²⁰. Thus, we can consider p being very low as alleged in the literature, somewhere in-between **0.2** to

¹⁷A more than 79.5 percent change in output may be unlikely. Historical data indicate that the Philippines’ nominal output decreased by the maximum of 11.8%, while the Thailand’s and the Korea’s real output decreased by 17.35% and 20.95%, respectively, after the Asian currency crisis since 1997 (The data source is the IMF-IFS online).

¹⁸This depends on how to define a crisis. His definition is the monthly percentage change in the exchange rate is larger than some threshold level *but* the average absolute percentage change in the 12 months before the period is smaller than some threshold level. 75 attacks were successful out of the total of 192 episodes, and thus the probability of successful crisis is approximated to 0.39.

¹⁹Those ten Asian countries include Indonesia, Korea, Malaysia, Philippines, Thailand, Turkey, India, Pakistan, Sri Lanka, and Singapore. For the rest of eleven non-Asian countries, the average probability of crisis is even lower, 0.22.

²⁰See Ito and Orri ([35]: p. 20), Figure 5, Panel A. The average out-of-sample predicted probability for 1997 is calculated by their benchmark model. The five crisis-hit countries are Indonesia, Malaysia, Korea, Philippines, and Thailand.

0.3²¹.

Thus, in order to determine the sign of Equation (18) with deeper confidence, we need to know the configuration of underlying parameters α and p , in addition to the assumptions of $\sigma = 1$ and $\phi = 2$. **Figure 2** depicts the bracket term of (18) that determines its sign as a second-order function of α for possible values of p . If this bracket term of (18) is positive, this country gains by starting to accumulate foreign reserves from the initial zero level of reserves.

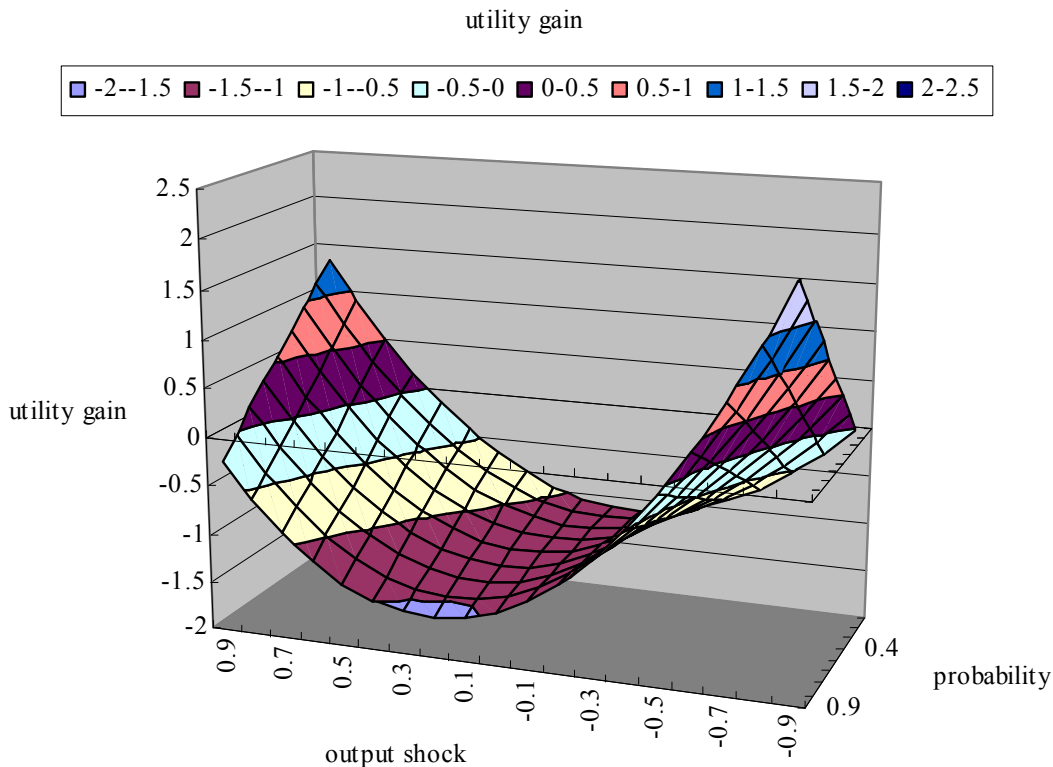
As observed from the Figure, the country with loss aversion (reflected in $\sigma = 1$) gains when a negative output shock is relatively large and the probability of bad state is higher. It also gains from reserve accumulation when a positive output shock is relatively large and the probability of good state is higher.

Since **Figure 2** is drawn with controlled σ and ϕ , the robustness must be examined with their different values. **Figure 3** is drawn with $\phi = 4$, meaning that the country is more risk averse in terms of the relative risk aversion à la Arrow and Pratt, retaining $\sigma = 1$ (same degree of loss aver-

sion)²².

Figure 3 reveals that, although the basic sign pattern is unchanged from the previous **Figure 2**, the depicted surface is noticeably lifted up, and as a result, the country gains from reserve accumulation for smaller output shocks. Thus, we deduce that a risk averse country has a good reason for reserve accumulation as expected.

The next parameter to be examined for robustness is the loss aversion ratio, defined as $(1 + \lambda_L)/(1 - \lambda_H)$, the extra weight attached to the utility for the bad state of nature against the extra weight attached to that for the good state of nature. Up to here we have assumed it to be 2, and hence $\sigma = 1$. Here, the first case we consider is to change $(1 + \lambda_L)/(1 - \lambda_H)$ to 4, meaning that the country becomes “twice” as more concerned with bad states while other things being equal. Using the definitions of λ_L and λ_H give in Appendix, σ is 3 in this case. And the second case is to change $(1 + \lambda_L)/(1 - \lambda_H)$ to 1.5, and σ in this case turns out to be 0.5. It should be mentioned that the basic characteristics of the sign pattern reflected by **Figure 1** are not significantly affected with those change in σ ²³.



A benchmark case: $\sigma = 1$ and $\phi = 2$.

Figure 2. Utility gains from foreign reserve accumulation.

²¹Which probability from the two methods, the “indicators” method or a probit model, is more reliable was also considered by Budsayaplakorn *et al.* [34]. Since the probit model is based on estimation, whether the probability calculated by the model is accurate is determined by the three criteria (*i.e.* the mean squared error, the quadratic probability score, and the log probability score). Formally, the outperformance of the probability using the probit model was confirmed by the Global Squared Bias criterion.

²²The coefficient of relative risk aversion is sometimes assumed larger than 2 (the Samuelson’s presumption). For example, King *et al.* [36] assumes $\phi = 5$.

²³The calculation results and the figures are available from the author on request.

Thus, it is apparent from **Figures 1 and 2** that the country gains from reserve accumulation a) for higher probability of bad state with higher negative output fluctuation, and also b) for higher probability of good state with higher positive output fluctuation. But the country loses by reserve accumulation when output shock is small, regardless of the probability of bad state. To confirm this relative irresponsiveness of change in utility with respect to loss aversion, we perform additional robustness exercise with a larger $\sigma = 7$, other things being equal. The result with this strong emphasis on loss aversion parameter, however, does not significantly affect the sign pattern depicted by **Figure 2 or 3**. This suggests that the country may not gain by simply adding a larger weight to the bad state in their utility function²⁴.

This finding may offer a new interpretation for huge foreign reserve accumulation by both China and Japan, although they have been very much different not only in political systems, but also economic aspects: China has

been a developing country with a high output growth rate, but their per capita income has still remained very low. On the other hand, Japan has been a member of the OECD with a higher per capita income, but it has been suffering from a stagnant economy (deflation) for more than twenty years²⁵. For these two countries with high accumulation of foreign reserves, our analysis offers a *consistent* explanation for the “optimality”.

For a remarkable increase in **Chinese** foreign reserve accumulation, it can be argued that, because of prolonged steady rate of economic growth, α (a change in output), is relatively large, while the probability of bad state (or crisis), p , is low, since China has been free from speculative attacks because of relatively strict capital controls²⁶. Thus, it has been “optimal” for China to accumulate huge foreign reserves from the standpoint of loss aversion. It is surprising to realize that almost opposite reasons may apply for large **Japanese** foreign reserve accumulation: Because of the “lost two decades” of economic slump, $-\alpha$

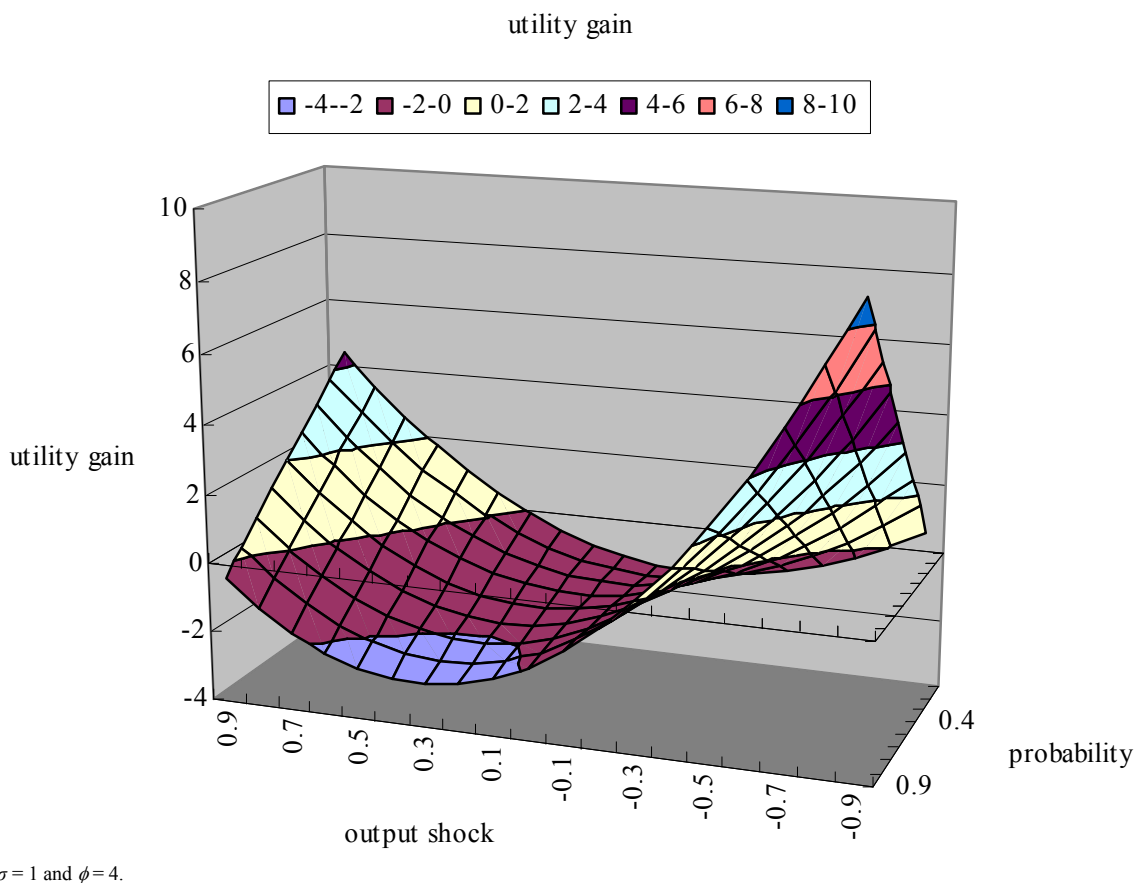


Figure 3. Utility gains from foreign reserve accumulation.

²⁴The most notable change from **Figure 2 or 3** is that the slope of the surface is steeper in this case. The calculation result and the Figure are also available on request.

²⁵The Chinese and the Japanese per capita incomes in 2010 were US\$4382.14 and US\$42820.39, respectively, according to the IMF's World Economic Outlook (April, 2011).

²⁶The Chinese average growth rate of the last decade (2002-2011) is 10.6 % (IMF-World Economic Outlook, April, 2011). Thus, China is an example of case (a).

(<0) is relatively large in absolute value, while p has been alleged to be large because of huge unsettled national debts, almost 600 trillion yen (more than 120 percent of nominal GDP) at the end of 2009²⁷. Thus, it may be reasonable to understand why Japan has such a large “optimal” accumulation of foreign reserves for a reason of loss aversion, even though this interpretation may be reasonable in hindsight once we recall that Japan has been blamed for attaining the unprecedented high rate of economic growth by increasing its exports like “concentrated heavy rain”, or by “mercantilism”.

The next question to ask is how much foreign reserves to accumulate as the “optimal” level. The answer has already been given in our discussion in Section 3, where we have found that the optimal precautionary savings as buffer stock should be larger than the expected loss (see Equation (9)) but smaller than the possible output loss (see Equation (5))²⁸.

6. Effects of Exogenous Changes on the Optimal Reserves

This section examines the effects of exogenous shifts in the four underlying parameters, and explores if a policy of holding of foreign reserves at autarky is much more legitimate and convincing. The results are summarized in Panel A of Table 1.

6.1. An Increase in p

The first exercise is the effect of an exogenous increase in the probability of bad state, p . Partial differentiation of (18) with respect to p yields:

$$\frac{\partial}{\partial p} \left(\frac{\partial U_1}{\partial R} \right) = \frac{\beta u'(Y_2) F'(K_2)}{(1 + p\sigma)^2} \{2\alpha(1 + \sigma)(1 - \phi)\} \quad (19)$$

which is negative for $\phi = 2$ and $\sigma = 1$. In other words, when the probability of a bad state is more likely to increase, the desirability of holding foreign reserves for a precautionary purpose is diminished. This is plausible,

because if the country holds foreign reserves, the total spending would be reduced in a worse situation under the risk of a higher probability of a bad state. This characteristic is visualized clearly in Figure 2, where the utility surface has a slope downward with the probability of bad state, p .

However, another interpretation would be possible when the (Arrow-Pratt) measure of relative risk aversion ϕ is smaller than unity. This in fact is equivalent to a qualification made by Aizenman [19] (p. 943) that “the concavity of the marginal utility is playing only a secondary role”. Needless to say, (19) is also positive for $\phi = 0$, meaning that the desirability of holding foreign reserves is increased when the instantaneous (period) utility function is a linear function of consumption. If (19) is evaluated at the initial point where $\alpha = 0$ (i.e. no output uncertainty), the whole expression reduces to 0, implying that the desirability of holding foreign reserves is insensitive to the probability of state of the nature.

6.2. An Increase in σ

The second exercise is an effect of increase in a parameter which measures loss aversion of the policy authority of this country, i.e. σ . Partial differentiation of (18) with respect to σ yields:

$$\frac{\partial}{\partial \sigma} \left(\frac{\partial U_1}{\partial R} \right) = \frac{\beta u'(Y_2) F'(K_2)}{(1 + p\sigma)^2} \{2\alpha p(1 - p)(1 - \phi)\} \quad (20)$$

which is negative for $\phi = 2$. Thus, the desirability of holding foreign reserves diminishes for a higher measure of loss aversion. This is also plausible, as the partial derivatives of λ_H and λ_L in (17) with respect to σ are shown to be positive for the former, but zero for the latter for $p = 0.5$. Thus, for a higher measure of loss aversion σ , the country attaches a higher weight for a good state, but an unchanged weight for a bad state, implying that the country’s precautionary motive for holding foreign reserves is weaker than otherwise.

However, (20) is positive when $\phi < 1$. This implies that, if the preceding remark on the case (1) by Aizenman [19] is plausible, the desirability of holding foreign reserves increases for a higher measure of loss aversion. A similar interpretation to the preceding case (1) applies to this case when the instantaneous (period) utility function is closer to a linear function of consumption. Since a marginal increase in loss aversion for a linear utility function implies a decrease in the expected utility level, the desirability of holding foreign reserves increases than otherwise. Also, if (20) is evaluated at the initial point where $\alpha = 0$, the whole expression reduces to 0, i.e. the desirability of holding foreign reserves as a buffer stock is insensitive to loss aversion.

²⁷The Japanese average annual growth rate of the “lost two decades” (1992-2011) is only 0.87%. The lowest growth rate is -6.29% in 2009 (IMF-World Economic Outlook, April, 2011). According to Figure 2, utility is increased by reserve accumulation at a point where $p = 0.6$ and $\alpha = -0.6$. This “indifference curve” shows the same utility for a lower p with a higher α . A similar indifference relation is observed in Figure 3 where, for example, $p = 0.8$ with $\alpha = -0.3$. Thus, Japan is an example of case (b). According to the Bloomberg’s report dated April 21, 2011 (<http://www.bloomberg.com/news>), the “sovereign vulnerability index ranks Japan as the second most vulnerable”, next to Greece. The USA was ranked 5th.

²⁸According to the press release by the Japanese Cabinet Office dated June 24, 2011, the total loss incurred by the “Great East Japan Earthquake” on March 11, 2011 is estimated as 16.9 trillion yen, about 3.5% of the 2010 nominal GDP (479.2 trillion yen). The Japanese foreign reserve ratio to GDP was 19% at the end of 2010, while that of China was 49%. (Author’s calculation based on the IMF’s World Economic Outlook (April, 2011)).

Table 1. Comparative statics results for benefits of foreign reserves (R) and external debts (B).

Panel A		
Parameter	R (Foreign Reserves)	
p	$\frac{\partial}{\partial p} \left(\frac{\partial U_1}{\partial R} \right) = \frac{-\beta u_2'}{(1+p\sigma)^2} F'(K_2) (2\alpha(1+\sigma)(\phi-1))$	
σ	$\frac{\partial}{\partial \sigma} \left(\frac{\partial U_1}{\partial R} \right) = \frac{-\beta u_2'}{(1+p\sigma)^2} F'(K_2) [2\alpha p(1-p)(\phi-1)]$	
α	$\frac{\partial}{\partial \alpha} \left(\frac{\partial U_1}{\partial R} \right) = \frac{\beta u_2'}{(1+p\sigma)} F'(K_2) \{ [p(1+\sigma) - (1-p)](1-\phi) + 2\alpha\phi(1+p\sigma) \}$	
ϕ	$\frac{\partial}{\partial \phi} \left(\frac{\partial U_1}{\partial R} \right) = \frac{\beta u_2'}{(1+p\sigma)} F'(K_2) \{ \alpha [2(1-p) - (1-\alpha)(1+p\sigma)] \}$	
δ	-	
β	$\frac{\partial}{\partial \beta} \left(\frac{\partial U_1}{\partial B} \right) = \frac{-u_2'(Y_2)}{1+p\sigma} F'(K_2) \{ (1-\alpha\phi)(1+\alpha) + p[\sigma(1-\alpha)(1+\alpha\phi) + 2\alpha(\phi-1)] \}$	
Note: Evaluated at $R = B = 0$.		
Panel B		
Parameter	B (International Debts): No Defaulting	B (International Debts): Defaulting*
p	$\frac{\partial}{\partial p} \left(\frac{\partial U_1}{\partial B} \right) = \frac{\beta u_2'}{(1+p\sigma)^2} F'(K_2) [2\alpha(1+\sigma)(\phi-1)]$	$\frac{\partial}{\partial p} \left(\frac{\partial U_1}{\partial B} \right) = \frac{\beta u_2'}{(1+p\sigma)^2} 91 + \sigma [2\alpha(\phi-1)F'(K_2) + (1+\alpha\phi)]$
σ	$\frac{\partial}{\partial \sigma} \left(\frac{\partial U_1}{\partial B} \right) = \frac{\beta u_2'}{(1+p\sigma)^2} F'(K_2) [2\alpha p(1-p)(\phi-1)]$	$\frac{\partial}{\partial \sigma} \left(\frac{\partial U_1}{\partial B} \right) = \frac{\beta u_2'}{(1+p\sigma)^2} \{ p(1-p) [2\alpha(\phi-1)F'(K_2) + (1+\alpha\phi)] \}$
α	$\frac{\partial}{\partial \alpha} \left(\frac{\partial U_1}{\partial B} \right) = \frac{-\beta u_2'}{(1+p\sigma)} F'(K_2) \times \{ [p(1+\sigma) - (1-p)](1-\phi) + 2\alpha\phi(1+p\sigma) \}$	$\frac{\partial}{\partial \alpha} \left(\frac{\partial U_1}{\partial B} \right) = \frac{\beta u_2'}{(1+p\sigma)} \times \{ -F'(K_2) \{ [p(1+\sigma) - (1-p)](1-\phi) + 2\alpha\phi(1+p\sigma) \} + \phi p(1+\sigma) \}$
ϕ	$\frac{\partial}{\partial \phi} \left(\frac{\partial U_1}{\partial R} \right) = \frac{-\beta u_2'}{(1+p\sigma)} F'(K_2) \times \{ \alpha [2(1-p) - (1-\alpha)(1+p\sigma)] \}$	$\frac{\partial}{\partial \phi} \left(\frac{\partial U_1}{\partial R} \right) = \frac{\beta u_2'}{(1+p\sigma)} \{ -F'(K_2) \alpha [2(1-p) - (1-\alpha)(1+p\sigma)] + \alpha p(1+\sigma) \}$
δ	-	$\frac{\partial}{\partial \delta} \left(\frac{\partial U_1}{\partial B} \right) = \frac{-\beta u_2'}{(1+p\sigma)} [p(1+\sigma)] [\alpha\phi + (1-\alpha)(1+2\alpha\phi)F'(K_2)]$
β	$\frac{\partial}{\partial \beta} \left(\frac{\partial U_1}{\partial B} \right) = \frac{u_2'}{1+p\sigma} F'(K_2) \times \{ (1-\alpha\phi)(1+\alpha) + p[\sigma(1-\alpha)(1+\alpha\phi) + 2\alpha(\phi-1)] \}$	$\frac{\partial}{\partial \beta} \left(\frac{\partial U_1}{\partial B} \right) = \frac{u_2'}{1+p\sigma} \times \{ F'(K_2) [(1-\alpha\phi)(1+\alpha) + p[\sigma(1-\alpha)(1+\alpha\phi) + 2\alpha(\phi-1)]] + (1+\alpha\phi)(1+\sigma)p \}$

Note: Evaluated at $R = B = 0$. * means evaluated at $rB = 0$ and $\delta = 0$.

6.3. An Increase in α

The third comparative statics exercise is an effect of increase in future output shock, which implies that output is more uncertain as it fluctuates more widely. The increase in the fluctuation is reflected in an increase in α . Differentiating (18) partially with respect to α , and evaluating at the initial point where the historical level of foreign reserve accumulation is assumed zero and no output uncertainty ($\alpha = 0$) yields:

$$\begin{aligned} & \frac{\partial}{\partial \alpha} \left(\frac{\partial U_1}{\partial R} \right) \\ &= \frac{\beta u'(Y_2) F'(K_2)}{(1+p\sigma)} \quad (21) \\ & \times \{ [p(1+\sigma) - (1-p)](1-\phi) + 2\alpha\phi(1+p\sigma) \} \end{aligned}$$

which is ambiguous in sign, but positive for $\alpha > 0.083$ under our assumption of $\phi = 2$, $\sigma = 1$ and $p = 1/2$ at the autarky position. Thus, we arrive at a plausible prediction that the desirability of holding foreign reserves is likely to increase when future output becomes more uncertain than otherwise.

6.4. An Increase in ϕ

The fourth and the final exercise is an effect of increase in the attitudes towards risk, ϕ . Following a similar procedure as in the previous cases, partial differentiation of (18) with respect to ϕ yields:

$$\begin{aligned} & \frac{\partial}{\partial \phi} \left(\frac{\partial U_1}{\partial R} \right) \\ &= \frac{\beta u'(Y_2) F'(K_2)}{(1+p\sigma)} \{ \alpha [2(1-p) - (1-\alpha)(1+p\sigma)] \} \quad (22) \end{aligned}$$

Thus, if α is set to zero at autarky point, (22) is also zero. It is also positive for $\alpha > 1/3$ if evaluated under our assumptions of $\sigma = 1$, $\phi = 2$ and $p = 1/2$. The positive value seems plausible, as it implies that the desirability of holding foreign reserves is stronger for a higher relative risk aversion, ϕ . It is also clear that (22) is positive when $\sigma = 1$, $\phi = 2$ and $p = 1/3$, regardless of $\alpha(>0)$, implying that reserve holding becomes more desirable with relative risk aversion.

In addition to those effects, it should be mentioned that an increase in β , the subjective discount factor, will *ce-teris paribus* increase the desirability of holding the optimal level of foreign reserves for a precautionary purpose at the zero reserve level, on condition that (18) is unambiguously positive. This plausible implication is similar to the one in Aizenman and Marion [12,13]. Our new finding is that all effects summarized in Equations (20)-(22) are further strengthened by an increase in β .

7. The Optimal International Debts

This section considers the optimal level of B , and compares it with the optimal R . When the state of nature is good, the government has no reason for defaulting, but it is assumed that the government chooses to default when the state is bad. Thus, consumption in period 2 is changed to:

$$\begin{aligned} C_{2H} &= (1+\alpha)F(K_2) + (1+r_f)R - (1+r_B)B \\ &+ (I_1 + K_1) + (1+r)(Y_1 - C_1 - I_1 + B - R) \quad (23-1) \end{aligned}$$

$$\begin{aligned} C_{2L} &= (1-\delta)(1-\alpha)(K_2) + (1+r_f)R + (I_1 + K_1) \\ &+ (1+r)(Y_1 - C_1 - I_1 + B - R) \quad (23-2) \end{aligned}$$

where $(1 > \delta > 0)$ is “the additional loss of output in autarky, a common feature in sovereign debts models” (Alfaro and Kanczuk [25]: p. 25).

7.1. Without Defaulting

Following a similar procedure as we derived Equation (14) by assuming that the authority neither have external debts nor foreign reserves at the beginning of the first period, or alternatively the historical levels of external debts and foreign reserves are zero, the marginal benefit accruing from external debt holding is, for a non defaulting case:

$$\begin{aligned} \frac{\partial U_1}{\partial B} &= u'(Y_1) + \beta \{ (1-p)(1-\lambda_H)u'(Y_{2H})(r-r_B) \\ &+ p(1+\lambda_L)u'(Y_{2L})(r-r_B) \} \quad (24) \end{aligned}$$

The private sector of this economy is assumed to choose their optimal levels of consumption and investment *before* the monetary authority chooses B optimally. This implies that, for the optimal investment decision, I_1 , the same condition as Equation (13) is satisfied. Upon substitution of (13), λ_H , and λ_L , into (24) yields:

$$\begin{aligned} \frac{\partial U_1}{\partial B} &= \frac{\beta u'(Y_2)}{1+p\sigma} \\ & \times \{ (1-\alpha\phi)(1-p)[(1+\alpha)F'(K_2) - r_B] \\ & + (1+\alpha\phi)p(1+\sigma)[(1-\alpha)F'(K_2) - r_B] \} \quad (25) \end{aligned}$$

Evaluation of (25) with our assumption $r_B = 0$ (Aizenman and Marion [3,4]) reduces (25) to:

$$\begin{aligned} & \frac{\partial U_1}{\partial B} \\ &= \frac{\beta u'(Y_2) F'(K_2)}{1+p\sigma} \quad (26) \\ & \times \{ (1-p)(1-\alpha\phi)(1+\alpha) + p(1+\sigma)(1+\alpha\phi)(1-\alpha) \} \end{aligned}$$

Thus, at the autarky point where $p = 0.5$, $\sigma = 1$ and $\phi = 2$ as before, the optimal level of external debts is unambiguously positive for α in between 0 and 0.795. Thus, it

is optimal for the country to hold some external debts for this region of α . However, recalling that the optimal level of foreign reserves is zero for this region of α , we can see that this result is consistent with Alfaro and Kanczuk [25], arguing that the optimal level of foreign reserves is zero, while that of international debts is positive, based on their simulation. What we have found is that our model based on *loss aversion* also suggests that their conclusion is true for the extent of output uncertainty being in between $0 < \alpha < 0.795$. For larger uncertainty $\alpha \geq 0.795$, the opposite conclusion is deduced; the optimal level of B is zero, while that of R is positive.

7.2. With Defaulting

When the government chooses to default for a low output level due to a bad state, we can derive the marginal change at the autarky point, following a similar procedure as before and using (23-2):

$$\frac{\partial U_1}{\partial B} = \frac{\beta u'(Y_2)}{1 + p\sigma} \times \left\{ (1 - \alpha\phi)(1 - p)[(1 + \alpha)F'(K_2) - r_B] + [1 + \alpha\phi(1 - \delta)]p(1 + \sigma) \times [(1 - \alpha)(1 - \delta)F'(K_2) + 1] \right\} \quad (27)$$

Evaluating at the initial autarky position with our assumption $r_B = 0$ and, in addition $\delta = 0$ reduces (27) to (see the Equation (28) below):

This is not only positive as (26) with the same parameter values for $0 < \alpha < 0.795$, but also greater than (26), implying that a utility gain from external debts is larger for defaulting case than for non-defaulting case. However, this larger utility gain with defaulting should not be emphasized, as the country in the longer-run will have a larger cost of inability to borrow from the world capital market for sometime before regaining credibility.

7.3. Effects of Exogenous Changes on the Optimal Debts with or without Default

A similar procedure of comparative statics exercises for the optimal B yields the results summarized in Panel B of **Table 1**. Several characteristics are outstanding from it. The first and the most noticeable characteristic is the sym-

metric result between R and B for non-defaulting case (the first column of Panel B). However, this symmetric nature is a natural consequence of the budget constraint (11'), in which R and B enter the constraint at the first period with opposite signs. Thus, since the marginal benefits of holding R (Equation (18)) or B (Equation (26)) are evaluated at the beginning of the first period at $R = B = 0$, the comparative statics results of R and B must have the opposite signs for each other. In other words, they are "substitutes" each other in the sense of Alfaro and Kanczuk [25].

Secondly, as a comparison between defaulting (the first column of Panel B) and non-defaulting (the second column) cases reveals, the comparative statics values are larger in absolute value for the case of non-defaulting than defaulting (except the case of δ). This implies that the optimality of holding B is strengthened for a defaulting case for this one-shot game. Moreover, this observation is also consistent with that of Alfaro and Kanczuk [25] who observe that they are not "complete substitutes". We confirm this characteristic in our model incorporating loss aversion.

8. Precautionary Saving with Loss Aversion

This section considers the optimal precautionary saving with loss or disappointment aversion in a simple two-period dynamic model under uncertainty. We simplify the previous model in section IV with an assumption of $R = B = 0$, and by disregarding production, and hence investment. Outputs are assumed given exogenously, but the second-period endowment of output is stochastic by α as before, $Y_{2H} = Y_2 + \alpha$ and $Y_{2L} = Y_2 - \alpha$. Thus, the problem faced by the government is to maximize (10') with respect to S (saving), subject to the lifetime budget constraint $C_1 + C_2/(1+r) = Y_1 + Y_2/(1+r)$, where S is defined by $S = Y_1 - C_1$. The first-order condition is:

$$u'(Y_1 - S) = (1 - p)(1 - \lambda_H)u'\{Y_2 + \alpha + (1+r)S\} + p(1 + \lambda_L)u'\{Y_2 - \alpha + (1+r)S\} \quad (29)$$

where $1/(1+r)$ is assumed equal to the discount rate, β .

Expanding the marginal utilities around the neighborhood of $S = 0$ and $\alpha = 0$ by Taylor series and approximating them at the first and the second degree, it can be shown that (see the Equation (30) below):

$$\frac{\partial U_1}{\partial B} = \frac{\beta u'(Y_2)}{1 + p\sigma} \left\{ F'(K_2)[(1 - \alpha\phi)(1 + \alpha)(1 - p) + (1 + \alpha\phi)p(1 + \sigma)[(1 - \alpha)]] + (1 + \alpha\phi)p(1 + \sigma)(1 - \alpha) \right\} \quad (28)$$

$$dS = \frac{\left(\frac{\Omega_1}{1+r} \right) [(1 - p)(1 - \lambda_H) - p(1 + \lambda_L)] + \frac{1}{2} \left\{ \frac{u''\{Y_2 + (1+r)S\}}{u'(Y_1)} \right\} \alpha^2}{\Omega_1 + (1+r)\Omega_2} = \frac{\left(\frac{\Omega_1}{1+r} \right) \left(\frac{1 - 2p - p\sigma}{1 + p\sigma} \right) \alpha + \frac{1}{2} \left\{ \frac{u''\{Y_2 + (1+r)S\}}{u'(Y_1)} \right\} \alpha^2}{\Omega_1 + (1+r)\Omega_2} \quad (30)$$

where Ω_i is the coefficient of Arrow-Pratt absolute risk aversion at time i ($\Omega_i = -u''/u' > 0$). The second term on the numerator of the right-hand side is unambiguously positive (or non-negative), invoking the so-called *Arrow's hypothesis* of decreasing (or non-increasing) Ω_i , since it implies that $u''' > 0$. Thus, for a given positive second term, a sufficient, but not necessary, condition for savings to increase by an increase in uncertainty (α) is $1 - 2p - p\sigma > 0$. The equation $1 - 2p - p\sigma = 0$ is a hyperbola in the $p - \sigma$ plane as shown in **Figure 4**.

If the second, positive, term of (30) is negligibly small for a small α , the sign of (30) is dominated by the sign of $1 - 2p - p\sigma$ (Aizenman [19]). In the absence of loss aversion ($\sigma = 0$), this implies that saving increases for $p < 1/2$. But under loss aversion, it can be confirmed that saving increases for an even smaller probability of output loss; for example, when $\sigma = 1$ (Aizenman, 1998), the country saves more even for $p < 1/3$ for a precautionary purpose. As discussed earlier, we may be able to approximate p being somewhere in between 0.2 to 0.3 from preceding literature on the Early Warning System against currency crises. When $p \leq 0.3$ the sufficient condition for $dS > 0$ is satisfied for $\sigma < 1.3$. An important implication is that our sufficient condition depicted in **Figure 4** is likely to be satisfied, and thus, that the optimal precautionary saving under loss aversion is quite likely to be positive for plausible value of p according to empirical crisis episodes.

9. Conclusions

This paper considers a theory of the “optimal” level of foreign reserves, with particular emphasis on the benefits derived from precautionary holding of reserves. Starting from a simple self-insurance model, we elaborate our analytical model in an intertemporal framework, and

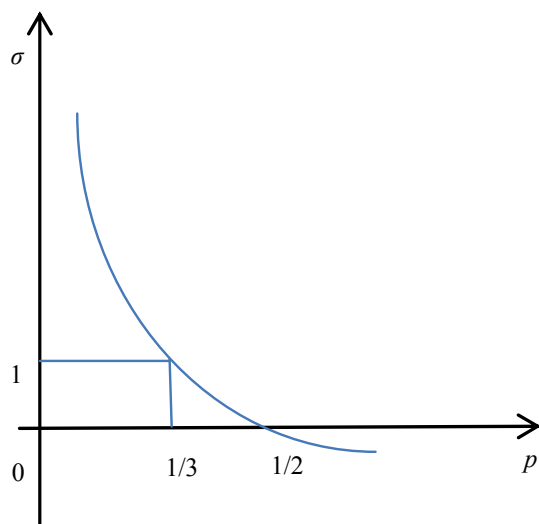


Figure 4. A sufficient condition for $dS > 0$.

presuppose that policy makers are motivated by neither financial stability nor financial mercantilism, but actually by holding precautionary reserves because agents are loss averse.

It is shown that the initial optimality condition for holding foreign reserves depends on the underlying parameters of the model. Given the loss aversion parameter and the coefficient of relative risk aversion, the utility surface depends on the probability of bad state and the degree of output uncertainty, as summarized by a convex surface depicted in **Figures 2** and **3**. Showing that the change in the welfare from the optimal level of foreign reserves under loss aversion depends crucially on the underlying parameter values (the probability of bad state, the Arrow-Pratt measure of relative risk aversion, the loss aversion rate, and the output uncertainty measure), we show how the present discounted level of welfare ($U1$) changes with configuration of the probability of bad state (p) and the output shock (α), controlling the rest of the two parameters. From the surface depicted in **Figures 2** in the $p-\alpha-U1$ space, we put forth our interpretation that huge accumulated foreign reserves observed in the actual historical data of China and Japan can be interpreted consistently within our intertemporal optimization model with loss aversion.

Several comparative statics exercises for the condition of the optimal foreign reserves are examined with respect to the underlying exogenous parameters. It is argued that the optimal level of foreign reserves decreases with the probability of bad state. The reason rests on the fact that the total spending decreases if the country saves more for a precautionary purpose. However, it is also clear that this prediction crucially depends on the degree of risk aversion. A similar dependency is also observed in another comparative statics exercise with respect to loss aversion. Plausible effects of output shocks and the attitudes toward risk are also derived.

Admitting that a country would not likely to make external borrowing for a precautionary purpose, we also consider such a possibility, partly because of theoretical completeness and partly because such a case has been considered in the previous literature. Allowing for a possibility of defaulting, we derive that similar optimality conditions with and without defaulting.

The optimal precautionary saving is also considered directly from our intertemporal optimizing model with loss aversion. We have shown that, as long as the *Arrow's hypothesis* of non-increasing absolute risk aversion, the optimal level of precautionary saving is likely to be positive with historically observed parameter values.

An important message drawn from the present investigation is that foreign reserve accumulation observed in the real world is consistent with the rational behavior of a country which has been concerned with loss aversion and

thus behaved optimally in a dynamic world with precautionary savings.

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Appendix

This section recapitulates the definition of loss aversion as put forth in Aizenman [19] and Azenman and Marion [12], and clarifies the demand side of the economy (*i.e.* expected utility) in the second period in more detail. The relationships between the degree of loss aversion (σ), the probabilities of bad and good states, ($p, 1 - p$), and the extra weights attached to utility in good and bad states (λ_H, λ_L) are presented. As it will be explained below, the definition of loss aversion is an application of the concept of risk aversion by subtracting the expected disappointment from the conventional expected utility.

Assume that the policy authority possesses the expected utility of uncertain consumption $\{C_{2s}\}$ in n states of nature, $s = 1, \dots, n$, denoted by $W(\sigma; \{C_{2s}\})$ in the second period. σ is called the loss aversion rate. Assume also that there is the "certainty equivalent" level of consumption μ , defined by $W(\sigma; \{C_{2s}\}) = u(\mu)$, where u is a conventional utility index with $u' > 0$ and $u'' < 0$. Then, loss aversion is defined by the existence of a positive parameter σ that satisfies:

$$\begin{aligned}
 & W(\sigma; \{C_{2s}\}) \\
 &= \int u(C_{2s}) f(C_{2s}) ds - \sigma \int_{C_{2s} < \mu} [u(\mu) - u(C_{2s})] \\
 & \quad \times f(C_{2s}) ds \tag{A-1} \\
 &= E[u(C_{2s})] - \sigma E[u(\mu) - u(C_{2s}) | C_{2s} < \mu] \\
 & \quad \times \Pr(C_{2s} < \mu)
 \end{aligned}$$

where f is the probability density function of C_{2s} , E is the expectation operator, and $\Pr(s)$ is the probability of state s . The term $E[u(\mu) - u(C_{2s}) | C_{2s} < \mu]$ is the expected value of $u(\mu) - u(C_{2s})$, conditional on C_{2s} below the cer-

tainty equivalent consumption, μ , *i.e.* the "expected disappointment". This average loss below the certainty equivalent consumption reflects the authority's sentiment of "disappointment" (Aizenman [19]: p. 935). Equation (A-1) postulates that the loss averse expected utility equals the difference between the conventional expected utility and a measure of loss aversion (σ) times the "expected disappointment".

Assume further that, for simplicity, there are only two states of nature, C_{2H} and C_{2L} , where C_{2H} corresponds to a higher and C_{2L} to a lower level of consumption with probability of $1 - p$ and p , respectively. Thus, p represents the probability of bad state. Then, since $u(\mu)$ does not depend on states of nature but a constant, it is straightforward to derive $W(\sigma)$. Upon integration of (A-1) yields:

$$\begin{aligned}
 W(\sigma) &= (1 - p)u(C_{2H}) + pu(C_{2L}) \\
 & \quad - \sigma p [W(\sigma) - u(C_{2L})] \tag{A-2}
 \end{aligned}$$

and thus solving (A-2) for $W(\sigma)$ yields:

$$\begin{aligned}
 & W(\sigma) \\
 &= (1 - p)(1 - \lambda_H)u(C_{2H}) + p(1 + \lambda_L)u(C_{2L}) \tag{A-3}
 \end{aligned}$$

where $\lambda_H \equiv p\sigma / (1 + p\sigma)$ and $\lambda_L \equiv (1 - p)\sigma / (1 + p\sigma)$. Note that λ_H and λ_L are non-linear with p , as in Tversky and Kahneman [27] and Benartzi and Thaler [30]. Note that they are both positive, $\lambda_H = \lambda_L$ for $p = 1/2$, and $\partial\lambda_H / \partial p > 0$ and $\partial\lambda_L / \partial p < 0$. Thus, the policy authority attaches a lower weight to $u(C_{2H})$, but a higher weight to $u(C_{2L})$ for a higher probability of bad state. Finally, it should be mentioned that $W(\sigma)$ reduces to the conventional expected utility when the loss aversion rate, σ , is zero.