Physical Review & Research International
4(2): 292-309, 2014

SCIENCEDOMAIN *international*
www.sciencedomain.org

Investigations of the Collective Properties of the Even Uranium Isotopes

S. B. Doma[1*] and H. S. El-Gendy[1*]

[1]*Faculty of Science, Alexandria University, Moharram Bay, Alexandria, Egypt.*

Authors' contributions

This work was carried out in collaboration between the two authors. Author SBD designed the study and the required nuclear models, wrote the protocol, and wrote the first draft of the manuscript. Author HSEG managed the analyses of the study, the computational work and the literature searches. The two authors read and approved the final manuscript.

Original Research Article

Received 25th June 2013
Accepted 2nd September 2013
Published 24th October 2013

ABSTRACT

The deformation structure of the even uranium isotopes is investigated in the framework of the collective model, the single-particle Schrödinger fluid model and the cranked Nilsson model. Accordingly, the rotational and vibrational energies, the nuclear moments of inertia, the total ground-state energy, the quadrupole moment, the Liquid Drop (L. D) energy, the Strutinsky inertia, the L. D. inertia, the volume conservation factor ω_0/ω_0^0, the smoothed energy, the BCS energy, the G-value and the transition probabilities of the exited-state in the ground-state band of the uranium isotopes: ^{232}U, ^{234}U, ^{236}U and ^{238}U have been calculated as functions of the deformation parameters β and γ, which vary in the ranges $(-0.50 \le \beta \le 0.50)$ and $(0^o \le \gamma \le 60^o)$. Moreover, a new formula that depends on the intrinsic quadrupole moment and the moment of inertia of the nucleus has been obtained to fit the transition probabilities of the exited-state in the ground-state band of the mentioned four isotopes.

Keywords: *Collective model; cranked Nilsson model; single-particle Schrödinger fluid; total energy; quadrupole moment; moment of inertia; transition probabilities.*

**Corresponding author: Email: sbdoma@yahoo.com, hselgendy1@yahoo.com;*

1. INTRODUCTION

In order to investigate the deformation structure of deformed nuclei, it is necessary to introduce some collective variables to describe the cooperative modes of motion. The simpler model is often called "the collective model" and the distorted shell model is often called "the unified model" [1-4]. Both represent collective effects but in different ways.

The moments of inertia, or equivalently the absolute values of the rotational energies and the quadrupole moments require knowledge of the fine details of the intrinsic nuclear structure. Consequently, the investigation of the nuclear moments of inertia and the quadrupole moments represent a sensitive check for the validity of the nuclear structure theories [5-7].

The quantum fluid [8] is considered to be completely transparent internally with respect to motion of the constituent particles and it receives disturbances solely by a way of surface deformations. The fact that the quantum fluid is nearly incompressible comes about, not by particle to particle push (as in the case of an ordinary liquid) but by more subtle means. It is capable of collective oscillations, but it is the wall which organizes these disturbances, not nucleon to nucleon interactions. Oscillations experience a damping, but the mechanism of the damping is unlike that encountered in ordinary liquids. The rotational properties of the quantum fluid are quite different from those of ordinary fluids.

Moreover, the study of the velocity fields for the rotational motion led to the formulation of the so-called the Schrödinger fluid [9, 10]. Since the Schrödinger-fluid theory is currently an independent particle model, the cranking model approximation for the velocity fields and the moments of inertia play the dominant role in this theory.

The pure Nilsson model cannot be used neither for the calculations of the total energies nor for the calculations of the shape of the energy surfaces at large deformation. Hence, Strutinsky [11] had suggested a renormalization procedure, which is the shell-correction method, and it became possible to calculate realistic potential energy surfaces. Therefore, the oscillator part of the deformation energy has been calculated within this model and replaced the smooth part by the liquid drop energy at the same deformation [12]. The systematic solutions for an axially symmetric deformed nucleus with β- and γ-vibrations, i.e. the rotation-vibration model, are later obtained by Faessler and Greiner [13-19].

The cranked Nilsson Strutinsky (CNS) model [20-26] is a theoretical approach that provides a good physical interpretation for the different properties of deformed nuclei, and at the same time it allows carrying out systematic and accurate calculations of the different properties of the deformed even-even nuclei.

We have recently applied the CNS-model, the single-particle Schrödinger fluid model and the nuclear superfluidity model in order to calculate the electric quadrupole moments and the moments of inertia of the even-even p- and sd-shell nuclei [27] and the obtained results are consistent with the available experimental data.

Furthermore, in a previous paper [28] we have applied the collective model to calculate the rotational and vibrational energies of the even-even ytterbium: ^{170}Yb, ^{172}Yb and ^{174}Yb, hafnium: ^{176}Hf, ^{178}Hf and ^{180}Hf and tungsten: ^{182}W, ^{184}W and ^{186}W nuclei. Also, the single-particle Schrödinger fluid model was applied to calculate the nuclear moment of inertia of the nine mentioned nuclei by using the rigid-body model and the cranking model. Furthermore, the CNS-model has been applied to calculate L. D. energy, the Strutinsky inertia, the L. D.

inertia, the volume conservation factor ω_0/ω_0^0, the smoothed energy, the BCS energy, the G-value, the total ground-state energy and the quadrupole moment of the nine mentioned nuclei as functions of the deformation parameters β and γ, which are assumed to vary in the ranges $(-0.50 \le \beta \le 0.50)$ and $(0^o \le \gamma \le 60^o)$. Moreover, two polynomials in β are fitted to obtain formulas, which produce results that are consistent with the corresponding CNS-values for the total ground-state energy and the quadrupole moment of the mentioned nine nuclei.

In the present paper we applied the three models, as in the previous paper [28], to calculate the same properties for the four even-even uranium isotopes: ^{232}U, ^{234}U, ^{236}U and ^{238}U as functions of the deformation parameters β and γ, which are assumed to vary in the ranges $(-0.50 \le \beta \le 0.50)$ and $(0^o \le \gamma \le 60^o)$. Furthermore, the transition probabilities of the exited-state in the ground-state band of the uranium isotopes: ^{232}U, ^{234}U, ^{236}U and ^{238}U have also been calculated. Moreover, a new simple formula is obtained to produce results that are in good agreement with the corresponding experimental transition probabilities of the exited-state in the ground-state band of the mentioned four isotopes.

2. TRIAXIAL DEFORMED CRANKED NILSSON STRUTINSKY METHOD

In the triaxial deformed CNS method, the nucleons are assumed to move in a CNS potential [23-26] with the deformation being described by the deformation parameters β and γ. The cranking is performed around one of the principal axes: the z-axis, and the cranking frequency is given by ω. In these calculations the triaxial CNS model is used in the rotating frame. The model provides a microscopic description of the influence of rotation on single-particle motion. The rotation is treated classically and the nucleons are considered as independent particles moving in an average rotating potential. The basic developments leading to the modified single-particle oscillator potential are described in [3, 20, 29, 30], while cranking was introduced in [21, 28]. The used single-particle Hamiltonian in this model, H^{ω}, is given in [31-33]. The diagonalization of the Hamiltonian H^{ω} gives the eigenvalues e_i^{ω} and the eigenvectors χ_i^{ω}. Furthermore, the single-particle energies in the laboratory system and the single-particle spin contributions m_i are also obtained.

The total energy is then obtained as

$$E = \sum_i e_i + E_c = \sum_i e_i^{\omega} + \hbar\omega \sum_i m_i + E_c\,, \tag{2.1}$$

with the total spin given by

$$I = \sum m_i\ . \tag{2.2}$$

These sums should be carried out over the occupied states where the occupation is determined from the order of the quantities e_i^{ω}. E_c in eq. (2.1) is the nuclear Coulomb energy which depends on deformation.

In the calculation of the total energy, the nuclear Coulomb energy should be treated as a residual force for the particles moving in the single-particle potential of this model. However, the most accurate procedure is very cumbersome; therefore one can determine the Coulomb energy of a homogeneous proton distribution with an ellipsoidal shape. The exact expression for the Coulomb energy of an ellipsoid E_c in units of Coulomb energy of a sphere $E_c^{(0)}$ was derived by Pal [34], Gob et al. [35] and Leander [36].

In order to overcome the difficulties encountered in the evaluation of the total energy for large deformation through the summation of the single particle energies, the Strutinsky shell correction method is adapted to $I \neq 0$ cases by suitably tuning the angular velocities to yield fixed spins [37, 38]. The methods of calculating the deformation characteristics of deformed nuclei are explained in [28].

3. The Single- Particle Schrödinger Fluid

According to the semi classical approach of dealing with the motion of the nucleon inside the nucleus, it is assumed that each nucleon in the nucleus is moving in a single- particle potential $V(r,\alpha(t))$, which is deforming with time t, through its parametric dependence on a classical shape variable $\alpha(t)$. Thus the Hamiltonian for the present problem is given by [9, 10]

$$H(\mathbf{r}, p;\alpha(t)) = \frac{p^2}{2m} + V(\mathbf{r},\alpha(t)). \tag{3.1}$$

The single-particle wave function $\psi(\mathbf{r};\alpha(t),t)$, which describes the motion of a nucleon, satisfies the time-dependent Schrödinger equation

$$H(\mathbf{r}, p;\alpha(t))\psi(\mathbf{r};\alpha(t),t) = i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r};\alpha(t),t). \tag{3.2}$$

We use polar form of the wave function and isolate the explicit time dependence in $\psi(\mathbf{r};\alpha(t),t)$ by an energy phase factor, i.e. we write [9]

$$\psi(\mathbf{r};\alpha(t),t) = \psi(\mathbf{r};\alpha(t))\exp\left\{-\frac{i}{h}\int_0^t \varepsilon(\alpha(t'))dt'\right\}, \tag{3.3}$$

where $\varepsilon(\alpha(t))$ is the intrinsic energy of the nucleon that depends on time through α. Then, the complex wave function $\psi(\mathbf{r};\alpha(t))$ is written in polar form as follows

$$\psi(\mathbf{r},\alpha(t)) = \Phi(\mathbf{r},\alpha(t))\exp\left\{-\frac{iM}{\hbar}S(r;\alpha(t))\right\}, \tag{3.4}$$

where $\Phi(\mathbf{r},\alpha(t))$ and $S(\mathbf{r},\alpha(t))$ are assumed to be real functions of $\mathbf{r}$ and α.

The average potential field is assumed to be in the form of a harmonic oscillator potential. The intrinsic energy of the single particle state is then,

$$E_{n_x n_y n_z} = \hbar\omega_x\left(n_x + n_y + 1\right) + \hbar\omega_z\left(n_z + 1\right). \tag{3.5}$$

For the frequencies ω_x , ω_y and ω_z in equation (3.5) we used the corresponding ones given by Nilsson [3], which are given by

$$\omega_z^2 = \omega_0^2\left(1 - \frac{4}{3}\delta\right), \tag{3.6}$$

$$\omega_x^2 = \omega_y^2 = \omega_0^2\left(1 + \frac{2}{3}\delta\right). \tag{3.7}$$

Accordingly, the condition of constant volume of the nucleus is guaranteed. The parameter δ is related to the well known deformation parameter β by

$$\delta = \frac{3}{2}\sqrt{\frac{5}{4\pi}}\beta . \tag{3.8}$$

By applying the time-dependent perturbation method and using the equation arising from the first-order perturbation of the wave function, we can calculate the first-order time-dependent perturbation correction to the wave function explicitly as function of the numbers of quanta of excitations corresponding to the Cartesian coordinates and the quantity σ , defined by [9]

$$\sigma = \frac{\omega_y - \omega_z}{\omega_y + \omega_z} , \tag{3.9}$$

which is a measure of the deformation of the potential.

We now examine the cranking moment of inertia in terms of the velocity fields. Bohr and Mottelson [2, 4] show that for harmonic oscillator case at the equilibrium deformation, where

$$\frac{d}{d\sigma}\sum_{i=1}\left(E_{n_x n_y n_z}\right)_i = 0 , \tag{3.10}$$

the cranking moment of inertia is identically equal to the rigid moment of inertia:

$$\Im_{cr} = \Im_{rig} = \sum_{i=1} m\left\langle y_i^2 + z_i^2 \right\rangle . \tag{3.11}$$

We note that the cranking moment of inertia $\Im_{cr}$ and the rigid moment of inertia $\Im_{rig}$ are equal only when the harmonic oscillator is at the equilibrium deformation. At other deformations, they can, and do, deviate substantially from one another [10].

The following expressions for the cranking-model and the rigid-body model moments of inertia, $\Im_{cr}$ and $\Im_{rig}$, are obtained [10]:

$$\Im_{cr} = \frac{E}{\omega_0^2}\left(\frac{1}{6+2\sigma}\right)\left(\frac{1+\sigma}{1-\sigma}\right)^{\frac{1}{3}}\left[\sigma^2(1+q)+\frac{1}{\sigma}(1-q)\right], \tag{3.12}$$

$$\Im_{rig} = \frac{E}{\omega_0^2}\left(\frac{1}{6+2\sigma}\right)\left(\frac{1+\sigma}{1-\sigma}\right)^{\frac{1}{3}}\left[(1+q)+\sigma(1-q)\right], \tag{3.13}$$

where E is the total single particle energy, given by (3.5) and q is the ratio of the summed single particle quanta in the *y*-and *z*-directions

$$q = \frac{\sum_{occ}(n_y+1)}{\sum_{occ}(n_z+1)}. \tag{3.14}$$

q is known as the anisotropy of the configuration.

4. The New Rotational and Vibrational Formulas

By analyzing the well known experimental rotational energy levels of the even-even deformed nuclei in the high mass region, we derived a new formula for the rotational energy levels, that depends upon the total spin momentum I and the nuclear moment of inertia $\Im$ in the following simple form [28]

$$E(\mathrm{I}) = \frac{A\mathrm{I}(\mathrm{I}+1)}{\left[1+\frac{D\mathrm{I}(\mathrm{I}+1)}{1-C\mathrm{I}(\mathrm{I}+1)}\right]}. \tag{4.1}$$

Here, A is the reciprocal-moment of inertia of the nucleus, $\mathrm{A} = \frac{\hbar^2}{2\Im}$. The value of A has been determined for all the considered isotopes by using the concept of the single-particle Schrödinger fluid [10].

Accordingly, our formula contains two parameters beside the nuclear moment of inertia. In our fitting, we determined C and D by inserting two values of the experimental rotational energies for middle values of I. In our calculations, we considered $\mathrm{I}=10$ and $\mathrm{I}=12$ to determine C and D.

When $D=C$ formula (4.1) gives the following simple relation

$$E(\mathrm{I}) = A\mathrm{I}(\mathrm{I}+1) - B\mathrm{I}^2(\mathrm{I}+1)^2, \tag{4.2}$$

where $B = AC$ This special case coincides with the AB-formula [40].

Accordingly, our new formula modifies the AB- formula by the correction factor:

$$\frac{1}{\{1+(D-C)[\mathrm{I}(\mathrm{I}+1)]\}}\ .$$

Furthermore, by analyzing the well known experimental β-band energy levels of the even-even deformed nuclei in the high mass region, we derived a new formula for the β-band energy levels, that depends on the total spin momentum I, the head band of β and the head band of γ and the nuclear moment of inertia $\Im$ in the following simple form [28]

$$E_{\beta-band}=-\frac{\hbar^2}{\Im}\frac{1}{g}\mathrm{I}^2(\mathrm{I}+1)^2+\frac{\hbar^2}{2\Im}\mathrm{I}(\mathrm{I}+1)+E_\beta\ . \tag{4.3}$$

Also, by analyzing the well known experimental γ-band energy levels of even-even deformed nuclei in the high mass region, we have derived a new formula for the γ-band energy levels that depends upon the total spin momentum I, the head band of γ, the number of neutrons N and the nuclear moment of inertia $\Im$ in the following simple form [28].

$$E_{\gamma-band}=\frac{\hbar^2}{2\Im}\frac{1}{N}\mathrm{I}^2(\mathrm{I}+1)^2+\frac{1}{2}\frac{\hbar^2}{2\Im}\mathrm{I}(\mathrm{I}+1)+E_\gamma\ . \tag{4.4}$$

The quantity g, appearing in equation (4.3) is given by:

$E_\gamma=g\hbar\omega_\gamma$ where $\hbar\omega_\gamma$ is the unit energy associated with E_γ.

5. Transition Probabilities of the Ground-State Band

It is well known that absolute gamma ray transition probabilities offer the possibility of a very sensitive test of nuclear models, and the majority of information regarding the nature of the ground state has come from studies of the energy level spacing. The transition probability values of the exited-state in the ground-state band constitute another source of nuclear information.

For coulomb excitation [7,39], the $B(E2)$ reduced transition probability in the case of a symmetric rotator (even-even nuclei) is given by

$$B(E2;\mathrm{I}\to\mathrm{I}+2)=\frac{5}{16\pi}e^2Q_0^2\left|\langle \mathrm{I}200\,|\,\mathrm{I}+2,0\rangle\right|^2. \tag{5.1}$$

By analyzing the well known experimental reduced transition probability, $B(E2)$, of even-even deformed nuclei in the high mass region we have derived a new formula which fits the values of $B(E2)$, and depends on the mass number A, the number of protons Z, the moment of inertia $\frac{\hbar^2}{2\Im}$ and the quadrupole moment Q_0, of the nucleus in the following form:

$$B(E2; \mathrm{I} \rightarrow \mathrm{I}+2) = \left\{ \sqrt{\frac{3A}{2Z}} |q_0| + \frac{1}{J} 0.498 \exp(|q_0|) \right\} e^2 r_0^2 , \qquad (5.2)$$

where $Q_0 = q_0 r_0^2$, $r_0 = 1$ fm, and $|q_0|$ is the absolute value of q_0. In equation (5.2) J is given by

$$\frac{\hbar^2}{2\Im} = JK \ , K = 1 \,\text{Kev}.$$

6. RESULTS AND DISCUSSION

We have calculated the reciprocal moments of inertia according to the cranking model and the rigid-body model of the single-particle Schrödinger fluid for the even-even deformed uranium isotopes; ^{232}U, ^{234}U, ^{236}U and ^{238}U as functions of the deformation parameter β, which is allowed to vary in the range from -0.50 to 0.50 with a step equals 0.01.

In Table-6.1 we present the best values of the reciprocal moments of inertia by using Schrödinger fluid for the even-even deformed isotopes: ^{232}U, ^{234}U, ^{236}U and ^{238}U. The values of the deformation parameter β, which produce the best values of the moments of inertia, are also given in this table. The corresponding experimental values are given in the last column [41-44].

Table 6.1 Reciprocal moments of inertia by using Schrödinger fluid for the even-even deformed isotopes: ^{232}U, ^{234}U, ^{236}U and ^{238}U

Nucleus	Reciprocal moments of inertia in KeV			
	β	$\frac{\hbar^2}{2\Im_{rig}}$	$\frac{\hbar^2}{2\Im_{cr}}$	$\frac{\hbar^2}{2\Im_{\exp}}$
^{232}U	0.19 -0.20	7.61 8.09	8.27 8.32	8.28
^{234}U	0.19 -0.20	7.43 7.88	7.31 7.25	7.29
^{236}U	0.20 -0.21	7.33 7.80	7.55 7.49	7.57
^{238}U	0.19 -0.21	7.29 7.76	7.80 7.84	7.82

It is seen from Table 6.1 that the calculated values of the moments of inertia by using the cranking-model are in excellent agreement with the corresponding experimental values. It is also seen that there are two possible values of the deformation parameter for each nucleus which produce the best consistency, one of which is positive and the other is negative. As expected, the rigid-body values of the reciprocal moments of inertia fall within the range (80% -90%) of the corresponding experimental values.

Experimentally, it has been found that there is a strong correlation between the $B(E2)$ value of the first 2^+ state and its energy $E_{2_1^+}$ and it is illustrated in the following form [1]

$$E_{2_1^+} B\left(E2, 2^+ \to 0^+\right) \approx (25 \pm 8) \frac{Z^2}{A} \text{ [MeV e}^2\text{ fm}^4\text{]}, \tag{6.1}$$

where A is the mass number. This empirical relation holds for all the nuclei throughout the nuclear table. Moreover, the experimental moment of inertia $\frac{\hbar^2}{2\Im_{\exp}}$ can be found from the energy of the first 2^+ state of the rotational band, equation (4.1); $\Im_{\exp} \approx 3/E_2{}^+$ [MeV^{-1}]. Applying the empirical rule (6.1) and formula (5.1) for the $B(E2)$ value, we get a connection between the deformation parameter β and the moment of inertia

$$\Im_{\exp} = \frac{27}{80\pi^2} r_0^4 \frac{A^{4/3} \cdot \beta^2 \cdot Z^2 e^2}{E_{2^+} \cdot B(E2)} \approx \frac{\beta^2 \cdot A^{7/3}}{400} \text{ [MeV}^{-1}\text{]}. \tag{6.2}$$

From the above equation, we see that the existence of two values of the deformation parameter β in our results of the nuclear moment of inertia, one of which is positive and the other is negative, agrees with the experimental finding.

In the numerical calculations of the rotational energies of the even-even deformed isotopes: ^{232}U, ^{234}U, ^{236}U and ^{238}U, we have used our new formula, equation (4.1). Furthermore, we have also calculated the rotational energies by using the AB-formula [40], the Wrake-Khadikikar formula [45], Harris-formula [46], the variable moment of inertia-formula [47] and the ɑb-formula [48]. Among the previous five formulas the results obtained by using the ɑb-formula are to some extent better than those of the other four formulas. Accordingly, we present only in Table-6.2 the calculated values of the rotational energies of the mentioned four isotopes, for even values of the total angular momentum I in the interval from 2 to 20, by using the ɑb-formula and the new formula together with the available experimental values. The experimental values are taken from [41-44].

It is seen from Table 6.2 that the calculated values of the rotational energies of the four isotopes by using the new formula are in better agreement with the corresponding experimental ones than those obtained by using the ɑb-formula.

In Table 6.3 we present the β-band energies of the four isotopes: ^{232}U, ^{234}U, ^{236}U and ^{238}U as functions of the total spin I by using the new formula, equation (4.3). The experimental values are taken from [41-44].

In Table 6.4 we present the γ-band energies of the four isotopes: ^{232}U, ^{234}U, ^{236}U and ^{238}U as functions of the total spin I by using the new formula, equation (4.4). The experimental values are taken from [41-44].

It is seen from Tables 6.3 and 6.4 that the calculated values of the β-band and γ-band energies of the four isotopes by using the new formulas are in good agreement with the corresponding experimental values. From the obtained results we have seen that a new three-parameter formula for the rotational band of a well-deformed nucleus is suggested on the basis of the phenomenological Bohr Hamiltonian. In the derivation of this formula a small axial asymmetry and vibrational effects (including anharmonicity) have been taken into account. The consistency among the obtained results by using this formula and the observed ground-state bands is astonishingly excellent, which might imply that the formalism described here may have some validity.

In Table 6.5 we present the $B(E2)$ reduced transition probability of the four isotopes: ^{232}U, ^{234}U, ^{236}U and ^{238}U as functions of quadrupole moments and reciprocal moments of inertia by using the new formula, equation (5.2). The experimental values are taken from [41-44].

In Table 6.6 we present the calculated values of the L. D. Energy, the Strutinsky inertia, the L. D. inertia, the volume conservation factor ω_0 / ω_0^0, the smoothed energy, the BCS energy and the G-value of the four isotopes: ^{232}U, ^{234}U, ^{236}U and ^{238}U for values of the deformation parameter β, and the nonaxiality parameter γ, which produce good agreement with the corresponding findings.

Table 6.2 Rotational energies of the four even-even deformed isotopes: ^{232}U, ^{234}U, ^{236}U and ^{238}U as functions of the total spin I by using the αb-formula [48] and the new formula, equation (4.1). The experimental values are taken from [41-44].

Nucl.	Case	$E(I)$ in KeV									
		$I=2$	$I=4$	$I=6$	$I=8$	$I=10$	$I=12$	$I=14$	$I=16$	$I=18$	$I=20$
^{232}U	αb	47.40	156.34	323.13	542.65	809.03	1116.28	1458.65	1830.96	2228.65	2647.82
	New	47.42	156.26	322.56	541	805.8	1111.5	1453.4	1828.04	2232.71	2665.84
	Exp	47.57	156.56	322.6	541	805.8	1111.5	1453.7	1828.1	2231.5	2659.7
^{234}U	*αb*	43.18	142.65	295.51	497.65	744.3	1030.38	1350.95	1701.41	2077.5	2475.8
	New	43.47	143.32	296.06	497.04	741.2	1023.8	1340.8	1689.16	2066.6	2471.9
	Exp	43.5	143.35	296.07	497.04	741.2	1023.8	1340.8	1687.8	2063	2464.2
^{236}U	*αb*	45.15	149.48	309.71	522.52	783.08	1087.46	1427.69	1802.1	2205.36	2633.65
	New	45.2	149.41	309.75	522.24	782.3	1085.3	1426.9	1803.6	2212.3	2650.89
	Exp	45.24	149.48	309.78	522.24	782.3	1085.3	1426.3	1800.9	2203.9	2631.7
^{238}U	*αb*	44.78	148.1	307.28	518.52	777.27	1078.65	1417.8	1790.06	2191.18	2617.35
	New	44.84	148.19	307.15	517.8	775.7	1076.5	1416.3	1791.92	2200.9	2641.54
	Exp	44.91	148.41	307.21	517.8	775.7	1076.5	1415.3	1788.2	2190.7	2618.7

Table 6.3 β -band energies of the four isotopes: ^{232}U, ^{234}U, ^{236}U and ^{238}U, in KeV, as functions of the total spin I by using the new formula, equation (4.3). The experimental values are taken from [41-44].

Nucleus		0^+	2^+	4^+	6^+	E_β	E_γ
^{232}U	Exp.	691.42	734.57	833.07	984.9	691.42	850.23
	Cal.	691.42	740.34	849.04	1004.4		
^{234}U	Exp.	809.907	851.74	947.64	1096.12	809.907	912.14
	Cal.	809.907	853.19	949.70	1088.65		
^{236}U	Exp.	919.14	960.3	1050.85	…….	919.14	942.76
	Cal.	919.14	963.86	1063.73	1207.97		
^{238}U	Exp.	927.21	966.13	1056.38	…….	927.21	1044.63
	Cal.	927.21	973.47	1077.24	1228.47		

Table 6.4 γ -band energies of the four isotopes: ^{232}U, ^{234}U, ^{236}U and ^{238}U, in KeV, as functions of the total spin I by using the new formula, equation (4.4). The experimental values are taken from [41-44].

Nucleus		2^+	3^+	4^+	5^+	E_β	E_γ
^{232}U	Exp.	866.79	911.49	970.71	……	691.42	850.23
	Cal.	877.17	908.36	956.56	1027.44		
^{234}U	Exp.	926.72	968.425	1023.77	1090.89	809.907	912.14
	Cal.	935.92	963.41	1005.83	1068.12		
^{236}U	Exp.	957.90	1001.5	1058.8	1127.38	919.14	942.76
	Cal.	966.68	994.99	1038.61	1102.58		
^{238}U	Exp.	1060.27	1105.71	1167.99	1232	927.21	1044.63
	Cal.	1069.95	1099.12	1144.00	1209.67		

Table 6.5 we present the $B(E2)$ reduced transition probability of the four isotopes: ^{232}U, ^{234}U, ^{236}U and ^{238}U as functions of quadrupole moments and reciprocal moments of inertia by using the new formula, equation (5.2). The experimental values are taken from [41-44].

Nucleus	$\frac{\hbar^2}{2\Im_{cr}}$ in KeV	Q_{cal} in $barn^2$	$B(E2)_{cal}$ in e^2barn^2	$B(E2)_{\exp}$ in e^2barn^2
^{232}U	8.27	2.85	9.76	10
^{234}U	7.31	2.95	10.98	10.66
^{236}U	7.55	3.09	11.88	11.61
^{238}U	7.8	3.15	12.18	12.09

Table 6.6 The L. D. Energy, the Strutinsky inertia, the L. D. inertia, the volume conservation factor ω_0/ω_0^0, the smoothed energy, the BCS energy and the G-value of the four isotopes: ^{232}U, ^{234}U, ^{236}U and ^{238}U.

Nucl.	β	γ	L.D. energy MeV	Strutinsky inertia 1/MeV	L.D. inertia 1/MeV	ω_0/ω_0^0	smoothed energy MeV	BCS energy MeV	G-value MeV
^{232}U	-0.17	5^o	4.473	144.89	116.5	1.0053	3950.9	-2.61	0.089
^{234}U	-0.18	5^o	4.526	146.69	117.8	1.0075	2775.6	-2.14	0.089
^{236}U	-0.20	0^o	16.30	152.45	122.8	1.0134	2762.8	-1.61	0.088
^{238}U	-0.20	0^o	16.32	114.00	154.7	1.0135	2748.6	-1.50	0.088

In Figs 6.1, we present the variation of the total energy of the four mentioned nuclei as a function of the deformation parameters β and γ, which are assumed to vary in the ranges $(-0.50 \leq \beta \leq 0.50)$ on x-axis and $(0^o \leq \gamma \leq 60^o)$ on y-axis. The systematics in the structure of the calculated total energy as the neutron number varies through some uranium isotopes is shown in Fig. 6-1. The surface for ^{232}U is unstable more than the surfaces of the other isotopes ^{234}U, ^{236}U and ^{238}U with respect to the deformation parameter β, and the nonaxiality parameter γ. The considered ^{232}U, ^{234}U, ^{236}U and ^{238}U nuclei are prolate at $\gamma = 0$. On the other hand, in Figures 6-2 we present the variation of the quadrupole moments of the four mentioned nuclei as a function of the deformation parameters β and γ, which are assumed to vary in the ranges $(-0.50 \leq \beta \leq 0.50)$ on x-axis and $(0^o \leq \gamma \leq 60^o)$ on y-axis.

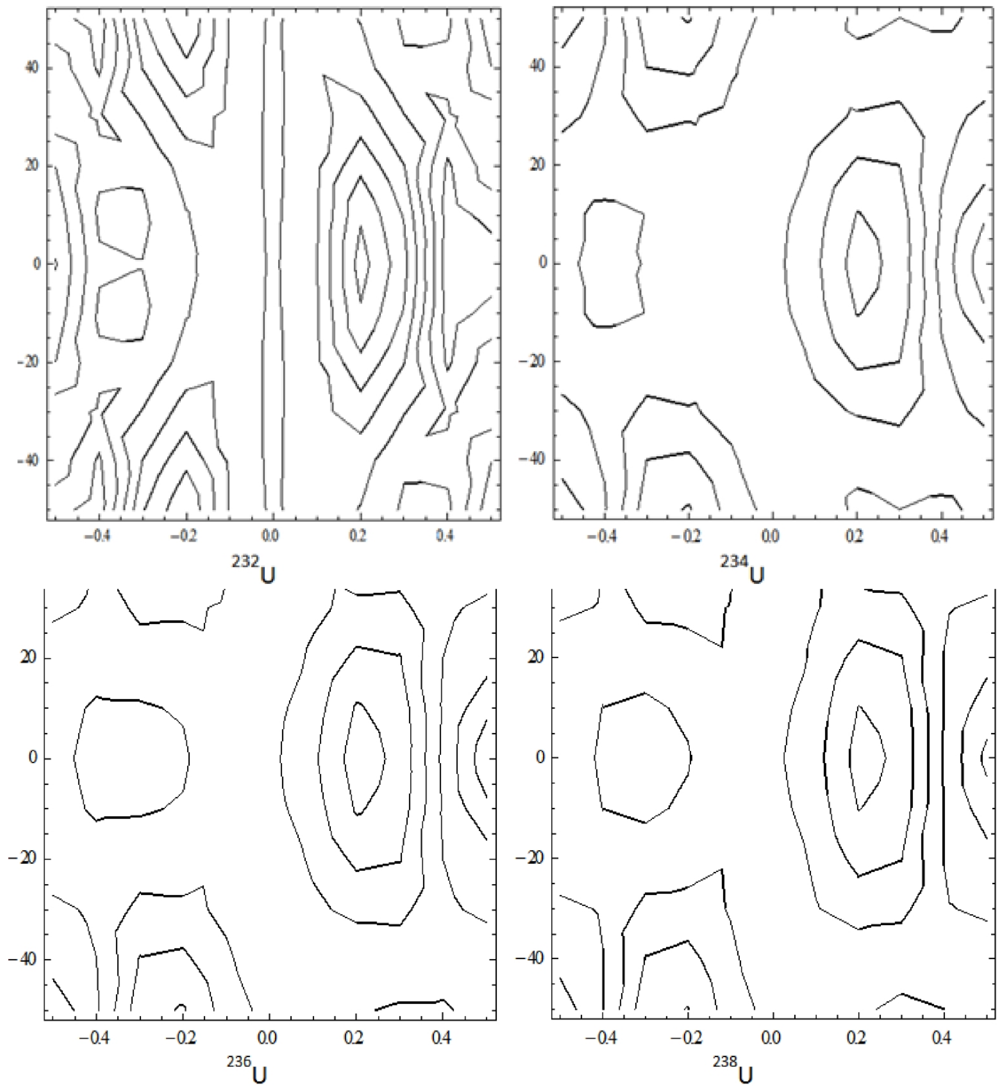

Fig. 6.1 Total energy for the uranium isotopes ^{232}U, ^{234}U, ^{236}U and ^{238}U, in KeV, as functions of the deformation parameters β and γ which are assumed to vary in the ranges $(-0.50 \le \beta \le 0.50)$ on x-axis and $(-50^{o} \le \gamma \le 50^{o})$ on y-axis.

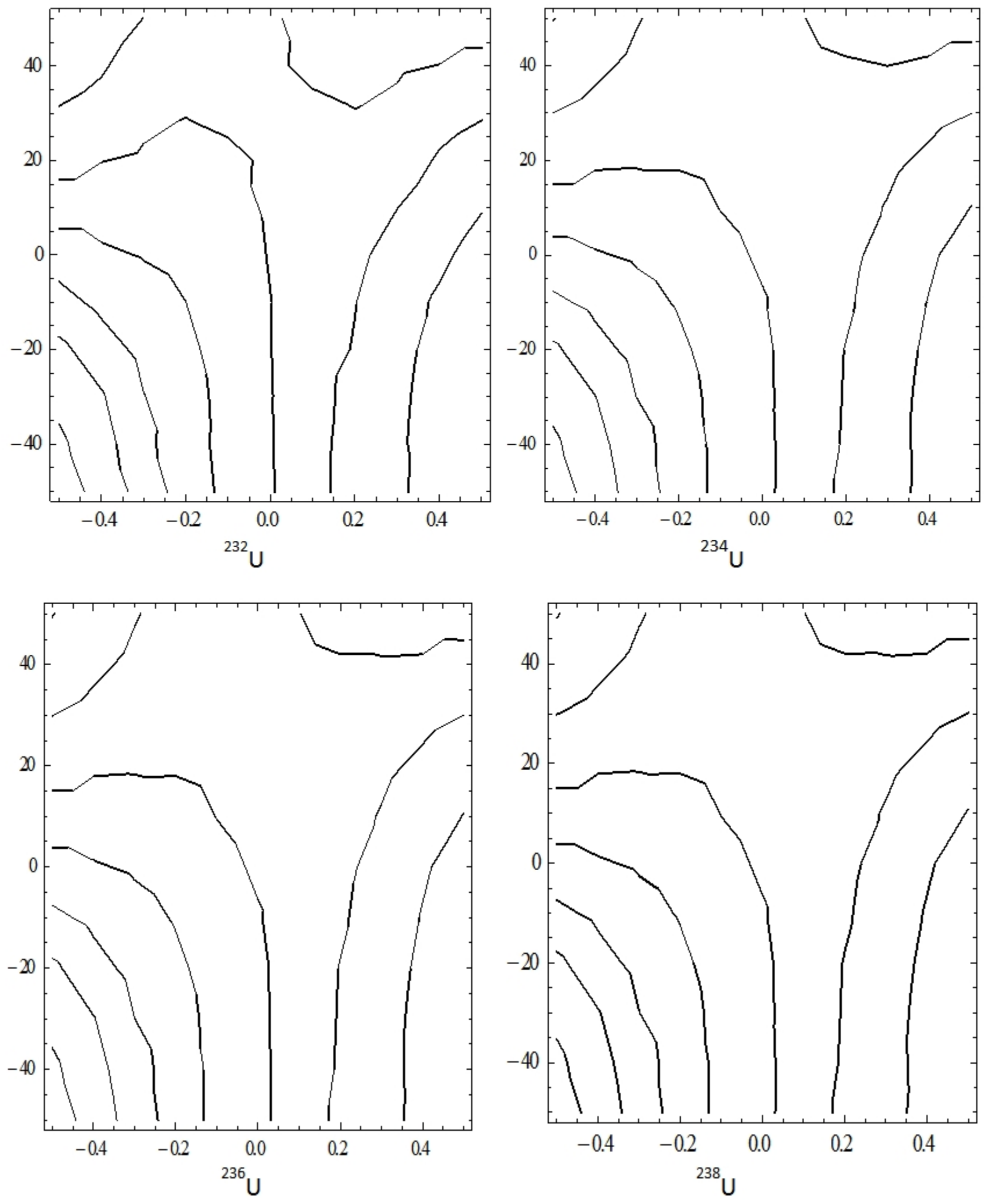

Fig. 6.2 Quadrupole moments for the ^{232}U, ^{234}U, ^{236}U and ^{238}U isotopes as functions of the deformation parameters β and γ which are assumed to vary in the ranges $(-0.50 \le \beta \le 0.50)$ on x-axis and $\left(-50^o \le \gamma \le 50^o\right)$ on y-axis.

It is important to note that the rotational energies, the β-band energies, the γ-band energies and the reduced transition probability $B(E2)$ of the four isotopes: ^{232}U, ^{234}U, ^{236}U and ^{238}U are all dependent on the nuclear moments of inertia, which have been calculated by applying the concept of the single- particle Schrödinger fluid. Accordingly, the study of the velocity fields for the rotational motion of the deformed nuclei, which has led to this concept, is very essential to produce results that are in good agreement with the corresponding experimental findings for the different characteristics of the deformed nuclei.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES

1. Ring P, Schuck P, The Nuclear Many-Body Problem, Springer-Verlag New York. 1980.
2. Bohr A , Mottelson B. Nuclear Structure, Word Scientific, Singapore. 1998;(2).
3. Nilsson SG, Danske Kg,. Videnskab. Binding states of individual nucleons in strongly deformed nuclei, Mat. Fys. 1955;29:(16).
4. Bohr A, Rotational motion in nuclei, Rev. Mod. Phys. 1976;48:365–374.
5. Davydov A S, Filippov G F, Rotational states in even atomic nuclei, Nucl. Phys. 1958; 8:237.
6. Davydov A S, Chaban A A, Rotation-vibration interaction in non-axial even nuclei, ibid. 1960; 20:499.
7. Judah M. Eisenberg and Walter Greiner, Nuclear Models (Nuclear Theory), Vol. 1, North-Holand Publishing Company, Amsterdam. 1970.
8. Hill D L and Wheeler J A, Nuclear Constitution and the Interpretation of Fission Phenomena, Phys. Rev. 1953; (89) 5:1102.
9. Kan K K, Griffin J J, Single-particle Schrödinger fluid. I. Formulation, Phys. Rev. C. 1977;15:1126
10. . Kan K K, Griffin J J, Independent particle Schrödinger fluid: moments of inertia, Nucl. Phys. A. 1978;301: 258
11. Strutinsky V M, Shell effects in nuclear masses and deformation energies, Nucl. Phys. A. 1967; 95:420.
12. Hai-Liang MA, Yu-Liang Yan, Zhang Xi-Zhen, Zhou Dong-Mei, Dong Bao-Guo, Spectroscopy and reduced transition probabilities of negative parity bands up to band termination in ^{45}Ti, Chinese Physics C, 33, Mar., 2009; 67-69
13. Faessler A, Greiner W, Die Rotations-Vibrations-Wechselwirkung in deformierten Atomkernen, Z. Phys., 1962;168:425.
14. Faessler A, Greiner W, Zur potentiellen Energie der γ -Schwingungen in deformierten gg-Kernen, Z. Phys., 1962;170;105.
15. Faessler A , Greiner W, Vergleich der Davydov-Theorie mit dem Modell des achsial-symmetrischen Kreisels mit Rotations-Vibrations-Wechselwirkung für deformierte gg-Kerne , Z. Phys., 1964; 177:190.
16. Faessler A. Einteilchen- und Kollektivanregung in deformierten Kernen ungerader Massenzahl, Nucl. Phys., 59:177 (1964).
17. Faessler A, Greiner W, Sheline RK, Magnetic properties of even nuclei, Nucl. Phys., 1965;80:417.
18. Faessler A, Greiner W and Sheline R K, The Coriolis anti-pairing and blocking effects in deformed even nuclei, Nucl. Phys. 1965; 62:241
19. Faessler A, Greiner W, Sheline R K, Rotation-Vibration-Interaction in Deformed Nuclei, Nucl. Phys. 1965;70:33.

20. Nilsson SG , Tsang C F, Sobiczewski A, Szymanski Z, Wycech S, Gustafsson C,. Lamm I L, Möller P, Nilsson B, On the nuclear structure and stability of heavy and superheavy elements, Nucl. Phys. 1969; A131:1.
21. Andersson CG, Larsson SE, Leander G, Möller P, Nilsson S G, Ragnarsson I, S. Äberg, Bengtsson R, Dudek J, Nerlo-Pomorska B, Pomorski K, Szymanski Z, Nuclear shell structure at very high angular momentum, Nucl. Phys. A, 1976; 268:205
22. Ragnarsson I, Nilsson S G, Sheline RK, Shell structure in nuclei, Phys. Rep., 1978; 45:1.
23. Huang Ya-Wei and Zhu Jian-Yu, Doubly magic properties in superheavy nuclei, Chinese Physics C. 2009; 33:92. (Supp. I).
24. Rajasekaran T R, Kanthimathi G. A comparative study of superdeformation in the A ≈ 150 and A ≈ 60 nuclei, Acta Physica Polonica B, 2008; 6:39
25. Vijayakumari G , Ramasubramanian V, Jacobi shape transition in stable Ti isotopes, Int. J. P. App. Scs., 2008;2(2):36.
26. Chandrasekaran Anu Radha, Velayudham Ramasubramanian and Emmanuel James Jebaseelan Samuel, Role of quadrupole deformation in proton emitting nuclei in the medium mass region, Turk. J. Phys. 2010;34:159
27. Doma S B,. Kharroube K A, Tefiha A D, El-Gendy HS, Applications of the Collective Model to Some Axially Symmetric Deformed Even-Even Nuclei, Alexandria Journal of Physics. 2011;1.
28. Doma S B, El-Gendy H S. Some deformation properties of the even even ytterbium, hafnium and tungsten nuclei, International Journal of Modern Physics E, 21, No. 9: 1250077. 2012.
29. Gustafsson C,.Lamm I L, Nilsson B, Nilsson S G, Nuclear deformabilities in the rare-earth and actinide regions with excursions off the stability line and into the superheavy region ,arkiv för Fysik, 1967;36:693
30. Bengtsson R, Larsson S E, Leander G, Möller P,. Nilsson SG, Ragnarsson I, Äberg S, Szymanski Z. Yrast bands and high-spin potential-energy surfaces, Phys. Lett. 1975; 57B:301.
31. Bengtsson T, Rotational bands and particle-hole excitations at very high spin, Nucl. Phys. A, 1985;436:14.
32. Larsson SE. The Nuclear Potential-energy Surface with Inclusion of Axial Asymmetry, Phys. Scripta, 1973; 8:17.
33. Larsson S E, Ragnarsson I , Nilsson SG, Fission barriers and the inclusion of axial asymmetry, Phys. Lett. 1972;38B: 269.
34. Pal M K, A semiclassical treatment of the dynamic ellipsoidal shapes in nuclei, Nucl. Phys. A. 1972;183:545
35. Gob U, Pauli HC, Alder K, Junker K, Ground state deformations in the rare-earth nuclei, Nucl. Phys. A. 1972;192:l
36. Leander G. The droplet model energy of axially asymmetric nuclei, Nucl. Phys. A, 1974;219: 245.
37. Anu Radha C,James E. Jebaseelan Samuel, Magicity from proton separation energies, DAE-BRNS Proceedings of Int. Symp. on Nucl. Phys. 2009;190:54.
38. Shanmugam G, Kalpana Sankar and Ramamurthi K, Structure of hot rotating s-d shell nuclei, Phys. Rev. C. 1995; 52:1443
39. Talmi I, Simple Models of Complex Nuclei: The Shell Model and Interacting Boson Model, Harwood Academic Publishers, U. S. A. 1993.
40. Guo-Mo Zeng, Iterative relationship in two-parameter formulas for rotational spectra and a universal equation, Phys. Rev. C. 1998;57(4):1726.
41. Browne E, Nuclear Data Sheets 107, 2006; 2579 – 2648. A = 232.
42. Browne E, Tuli JK, Nuclear Data Sheets 108, 2007; 681-772. A = 234.

43. Browne E, Tuli JK, Nuclear Data Sheets. 2006;107:2649-2714. A=236.
44. Chukreev FE, Makarenko VE, Martin MJ. Nuclear Data Sheets. 2002;97:129. A=238.
45. Warke CS, Khadkikar SB, Hartree-fock-bogoliubov projected spectra for finite nuclei, Phys. Rev. 1968;170:1041.
46. Harris SM, Large-spin rotational states of deformed nuclei, Phys. Rev. Lett. 1964; 13:663.
47. Mariscotti MAJ, Goldhaber GS, Buck B. Phenomenological analysis of ground-state bands in even-even nuclei, Phys. Rev. 1969;178:1864.
48. Holmberg P, Lipas PO. A new formula for rotational energies, Nuclear Phys. 1968; A117: 552.

Peer-review history:
The peer review history for this paper can be accessed here:
http://www.sciencedomain.org/review-history.php?iid=310&id=4&aid=2378