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Solving three Level Quadratic Programming Problem with Stochastic Parameters in the Objective Functions

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Abstract

This paper solves a stochastic three-level decision maker's model. This approach uses stochastic parameters in the objective function to solve a three level quadratic programming problem. The probabilistic nature of the objective functions is converted to an equivalence deterministic one and then a fuzzy programming technique will be used in each level to optimize its problem separately; implementing tolerance membership concept is used to generate Pareto optimal solution for these problems. Finally, the main results developed will be clarified by a numerical example in this paper.

Keywords: Stochastic programming; fuzzy programming; three level.

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1 Introduction

In the field of mathematical programming (MP), decentralized planning problems with multiple decision makers DMs were solved in a hierarchical decision making organization through developing multilevel programming (MLP) techniques. In a DM hierarchical organization, decision power is excuted sequentialy from higher to lower levels, where each unit or department independently optimizes its own benefit, through a conflicting environment of the other units actions. Government agencies, profit or nonprofit organizations, manufacturing plants, and logistic companies are all hierarchical organizations in which MLPP could be encountered [1,2,3,4,5,6].

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Fuzzy programming approaches applys tolerance membership concept to reach a fuzzy max-min decision model eventually generating Pareto optimal (satisfactory) solution for a three level programming problem. This implies that the lower level decision makers LLDM has to optimize their objective functions tolerating the preference of the upper level decision makers ULDM that was described by membership functions of fuzzy-set theory. The solution search is to be continued and membership functions are to be redefined if it didn't satisfy the ULDM preferences [2,6,7,8,9,10,11].

Decision problems of chance-constrained or stochastic optimization is applied when variables of an optimization model are defined in probabilistic quanatities instead of deterministic ones. Recently, there has been increasing importance for multiobjective stochastic optimization in solving real-life practical decision problems such as economics, water resource management, healthcare, transportation, agriculture, energy systems [12,13,14,15].

In Pramanik et al. [5] dealt with a chance constrained linear plus linear fractional bi-level programming problem through a fuzzy goal programming approach. In this paper some simple and easy techniques are used to solve the problem like the technique used in [5,14,16], converting the constraints probabilistic nature to equivalence deterministic constraints, using simplex method to give each decision maker's problem the best and the worst individual solution, the fuzzy set theory to formulate the previous results to membership function, and the tolerance concept to develop Tchebycheff problem generating Pareto optimal solution for our problem.

The paper is organized as follows: section 2 covers the problem formulation and solution concept of the model of three level quadratic programming problem with stochastic parameters in the objective function. converting the probabilistic nature of the stochastic nature in the objective functions to equivalence deterministic objective functions will be provided in section 3.The following section presents Fuzzy approach to solve the three level quadratic programming problem with stochastic parameters in the objective function. A numerical example to clarify the results will be discussed in section 5. The conclusion of this research is offered together with suggestions for future research directions in the final section.

2 Problem Formulation and Solution Concept

Let $x_j \in R^n, (j = 1,2,3)$ be a vector variable indicating the first decision level choice, the second decision level choice, and the third decision level choice, $n = \sum_{j=1}^{3} n_j$.

Let $F_i : R^n \to R^{N_i}, (i = 1,2,3)$ be the first level objective function, the second level objective function, and the third level objective function respectively. Let the FLDM (First Level Decision Maker), SLDM (Second Level Decision Maker), and TLDM (Third Level Decision Maker) have N_1 , N_2 and N_3 objective function, respectively.

Therefore, the three–level quadratic programming problem with stochastic parameters in the objective (TLQPSP) problem may be formulated as follows:

[First Level]

$$\max_{x_1} f_1(x) = sx + \frac{1}{2}x^T bx, \tag{1}$$

Where x_2, x_3 solves

[Second Level]

$$\max_{x_2} f_2(x) = sx + \frac{1}{2}x^T bx, \tag{2}$$

Where x_3 solves

[Third Level]

$$\max_{x_3} f_3(x) = sx + \frac{1}{2}x^T bx, \tag{3}$$

Subject to

$$(x_1, x_2, x_3) \in G. \tag{4}$$

Where

$$G = \{ x \in R^n : g_i(x) = \sum_{j=1}^{n} c_{ij} x_j \le d_i ,\ (i = 1,2,...,n) , x_1, x_2, x_3 \ge 0 \}. \tag{5}$$

Where f_1, f_2 and f_3 are the objective functions with stochastic parameters of the first level decision maker (FLDM), second level decision maker (SLDM), and third level decision maker (TLDM), b is $n \times n$ are real matrices contain random stochastic coefficients and s is $(1 \times n)$ matrix, the vector of decision variables x is n-vector partitioned between the three planners.

Definition 1.

For any $\left(x_1 \in G_1 = \{x_1 | (x_1, x_2, x_3) \in G\}\right)$ given by FLDM and $\left(x_2 \in G_2 = \{x_2 | (x_1, x_2, x_3) \in G\}\right)$ given by SLDM, if the decision-making variable $\left(x_3 \in G_3 = \{x_3 | (x_1, x_2, x_3) \in G\}\right)$ is the optimal solution of the TLDM, then (x_1, x_2, x_3) is the feasible solution of TLQP problem with stochastic parameters.

Definition 2.

If (x_1^*, x_2^*, x_3^*) is a feasible solution of the TLQP problem with stochastic parameters; no other feasible solution $(x_1, x_2, x_3) \in G$ exists; so (x_1^*, x_2^*, x_3^*) is the optimal solution of the TLQP problem with stochastic parameters.

3 Converting Probabilistic Nature of the objective functions to Equivalence Deterministic Objective Functions

The basic idea in this section is to convert the probabilistic nature of the objective function to equivalence deterministic objective function. Using:

$$f_r(x) = \sum_{j=1}^{n} s_{rj} x_j + k_1^r \sum_{j=1}^{n} x_j^2 E(b_j^r) + k_2^r \sqrt{\sum_{j=1}^{n} \frac{1}{2} x_j^2 \sigma^2 (b_j^r) x} , r = 1,2,...,k. \quad [14]. \tag{6}$$

Where $E(b_j^r)$ = mean of b_j^r and $\sigma^2(b_j^r)$ = variance of, and k_1^r , k_2^r are non-negative constants whose values indicate the relative importance of the mean and the standard deviation of the variable b_j^r for maximization. If k_1^r = k_2^r =1, it is an indication that equal importance is given to the maximization of the mean as well as the standard deviation of b_j^r .

So the TLQP problem can be written as:

[First Level]

$$\max_{x_1} f_1(x) = sx + \frac{1}{2}x^T b`x, \tag{7}$$

Where x_2, x_3 solves

[Second Level]

$$\max_{x_2} f_2(x) = sx + \frac{1}{2}x^T b`x, \tag{8}$$

Where x_3 solves

[Third Level]

$$\max_{x_3} f_3(x) = sx + \frac{1}{2}x^T b`x, \tag{9}$$

Subject to

$$(x_1, x_2, x_3) \in G. \tag{10}$$

Where

$$G = \{ x \in R^n : g_i(x) = \sum_{j=1}^{n} c_{ij} x_j \le d_i ,\ (i = 1,2,...,n) ,\ x_1, x_2, x_3 \ge 0 \}. \tag{11}$$

4 Fuzzy Approach of the Three Level Quadratic Programming Problem with Stochastic Parameters in the Objective Function

To solve the three–level quadratic programming problem with stochastic parameters in the objective function, one first convert the probabilistic nature of the objective functions to equivalence deterministic objective functions, after that gets the satisfactory solution that is acceptable to FLDM, and then every leader gives his decision variable and goal with some leeway to the his follower for him/her to seek the optimal solution.

4.1 FLDM Problem

First, the FLDM solves (7) subject to $\overline{G}$, by applying the simplex method. The individual best solution (F_1^*) and individual worst solution (F_1^-) of (7) subject to $\overline{G}$ are:

$$F_1^* = \underset{x \in \overline{G}}{Max}\, F_1(x) \quad , \; F_1^-(x) = \underset{x \in \overline{G}}{Min}\, F_1(x) \tag{12}$$

Goals and tolerances can then be reasonably set for individual solution and the differences of the best and worst solutions, respectively. This data can then be formulated as the following membership function of fuzzy set theory [8]:

$$\mu\,[F_1(x)] = \begin{cases} 1 & if \quad F_1(x) > F_1^*, \\ \dfrac{F_1(x) - F_1^-}{F_1^* - F_1^-} & if \quad F_1^- \le F_1(x) \le F_1^*, \\ 0 & if \quad F_1^- \ge F_1(x). \end{cases} \tag{13}$$

4.2 SLDM Problem

In the same way, the SLDM independently solves (8) subject to $\overline{G}$, by applying the simplex method. The individual best solution (F_2^*) and individual worst solution (F_1^-) of (8) subject to $\overline{G}$ are:

$$F_2^* = \underset{x \in \overline{G}}{Max}\, F_2(x) \quad , \; F_2^-(x) = \underset{x \in \overline{G}}{Min}\, F_2(x) \tag{14}$$

This information can then be formulated as the following membership function:

$$\mu\,[F_2(x)] = \begin{cases} 1 & if \quad F_2(x) > F_2^*, \\ \dfrac{F_2(x) - F_2^-}{F_2^* - F_2^-} & if \quad F_2^- \le F_2(x) \le F_2^*, \\ 0 & if \quad F_2^- \ge F_2(x). \end{cases} \tag{15}$$

4.3 TLDM Problem

In the same way, the TLDM independently solves (9) subject to $\overline{G}$, by applying the simplex method. The individual best solution (F_3^*) and individual worst solution (F_3^-) of (9) subject to $\overline{G}$ are:

$$F_3^* = \underset{x \in \overline{G}}{Max}\, F_3(x) \quad , \; F_3^-(x) = \underset{x \in \overline{G}}{Min}\, F_3(x) \tag{16}$$

This information can then be formulated as the following membership function:

$$\mu\,[F_3(x)] = \begin{cases} 1 & if \quad F_3(x) > F_3^*, \\ \dfrac{F_3(x) - F_3^-}{F_3^* - F_3^-} & if \quad F_3^- \le F_3(x) \le F_3^*, \\ 0 & if \quad F_3^- \ge F_3(x). \end{cases} \tag{17}$$

Now the solution of the three decision makers are disclosed. However, three solutions are usually different because of nature between three levels objective functions. The two level leaders know that using the optimal decisions, x_1^F, x_2^S as a control factor for the TLDM are not practical. It is more reasonable to have some tolerance that gives the TLDM an extent feasible region to search for his/her optimal solution, and reduce searching time or interactions.

In this way, the range of decision variable of the FLDM, SLDM with maximum tolerance t_1, t_2 and the following membership function specify x_1^F, x_2^S as:

$$\mu_{x_1}(x_1)=\begin{cases}\dfrac{x_1-(x_1^F-t_1)}{t_1} & x_1^F-t_1\le x_1\le x_1^F,\\ \dfrac{(x_1^F+t_1)-x_1}{t_1} & x_1^F\le x_1\le x_1^F+t_1.\end{cases} \tag{18}$$

$$\mu_{x_2}(x_2)=\begin{cases}\dfrac{x_2-(x_2^S-t_2)}{t_2} & x_2^S-t_2\le x_2\le x_2^S,\\ \dfrac{(x_2^S+t_2)-x_2}{t_2} & x_2^S\le x_2\le x_2^S+t_2.\end{cases} \tag{19}$$

Finally, in order to generate the satisfactory solution, which is also the optimal solution with overall satisfaction for all decision - makers, the Tchebycheff problem can be solved as the following [17]:

$$Max\,\delta, \tag{20}$$

Subject to

$$\frac{[(x_1^F+t_1)-x_1]}{t_1}\ge \delta I,$$

$$\frac{[x_1-(x_1^F-t_1)]}{t_1}\ge \delta I,$$

$$\frac{[(x_2^F+t_2)-x_2]}{t_2}\ge \delta I,$$

$$\frac{[x_2-(x_2^F-t_2)]}{t_2}\ge \delta I,$$

$$\mu[F_1(x)]\ \ge\ \delta,$$

$\mu[F_2(x)] \geq \delta,$

$\mu[F_3(x)] \geq \delta,$

$(x_1, x_2, x_3) \in G,$

$t_1, t_2 > 0,$

$\delta \in [0,1].$

Where δ is the overall satisfaction, and I is the column vector with all elements equal to 1s.

5 Numerical Example

To demonstrate the solution for TLQSPP, let us consider the following example:

[First Level]

$$\underset{x_1}{Max}\ F_1(x_1, b_1) = \underset{x_1,}{Max} \quad (2+b_1)x_1^2 + 2x_2^2 + x_3^2,$$

Where x_2 solves

[Second Level]

$$\underset{x_2}{Max}\, F_2(x_2, b_2) = \underset{x_2}{Max} \quad 2x_1 + (b_2+1)x_2^2 + x_3^2,$$

Where x_3 solves

[Third Level]

$$\underset{x_3}{Max}\, F_3(x_3, b_3) = \underset{x_3}{Max} \quad x_1^2 + x_2 + (b3+2)x_3^2,$$

Subject to

$$\{(x_1, x_2, x_3) | x_1 + x_2 + x_3 \leq 13,$$
$$x_1 + 2x_2 + x_3 \leq 17,$$
$$x_1 + x_2 + 2x_3 \leq 13.5,$$
$$x_1, x_2, x_3 \geq 0. \}.$$

Suppose that b_i, (i=1,2,3) is normally distributed random parameters with the following means and variances, E(b1)=3, E(b2)=5, E(b3)=3,VAR(b1)=4,VAR(b2)=16,VAR(b3)=4 .

1. In the first the probability nature of the objective functions will be converted to equivalent deterministic nature.
2. Secondly, reformulate the problem in to deterministic form as following:

[First Level]

$$\underset{x_1}{Max}\ F_1(x_1, b_1) = \underset{x_1,}{Max} \qquad 7x_1^2 + 2x_2^2 + x_3^2,$$

Where x_2 solves

[Second Level]

$$\underset{x2}{Max}\ F_1(x_2, b_2) = \underset{x_2}{Max} \qquad 2x_1 + 10x_2^2 + x_3^2,$$

Where x_3 solves

[Third Level]

$$\underset{x3}{Max}\ F_1(x_3, b_3) = \underset{x_3}{Max} \qquad x_1^2 + x_2 + 7x_3^2,$$

subject to

$$x_1 + x_2 + x_3 \le 13,$$

$$x_1 + 2x_2 + x_3 \le 17,$$

$$x_1 + x_2 + 2x_3 \le 13.5,$$

$$x_1, x_2, x_3 \ge 0.$$

First, the FLDM solves his/her problem as follows:

Find individual best and worst solutions by solving the FLDM problem using simplex method as $F_1^* = 1183$, and $F_1^-(x) = 0$.

Second, the SLDM solves his/her problem as follows: $F_2^* = 722.5$, and $F_2^-(x) = 0$.

Third, the TLDM solves his/her problem as follows: $F_3^* = 186.04$, and $F_3^-(x) = 0$.

Finally

1. Assume the FLDM control decision x_1^F is around (0) with tolerance 1.
2. Assume the SLDM control decision x_2^S is around (8.97) with the tolerance 2.

Max δ,

Subject to

$$5x_1^2 + 3x_2^2 + x_3^2 - 849\delta \geq 0,$$

$$x_1 + 5x_2^2 + x_3^2 - 402.30\delta \geq 0,$$

$$x_1^2 + x_2 + 4x_3^2 - 186.04\delta \geq 0,$$

$$x_1 + \delta \leq 14.03,$$

$$x_1 - \delta \geq 12.03,$$

$$x_2 + 2\delta \leq 10.970,$$

$$x_2 - 2\delta \geq 6.97,$$

$$\delta \in [0,1].$$

Whose compromise solution is $(x_1, x_2, x_3) = (13.02, 8.26, 3)$ and $\delta = 0.67$ (overall satisfaction for all decision maker's) and the objective function value are F1= 927.829, F2=90.247 , F3= 213.78 .

6 Summary and Concluding Remarks

A fuzzy approach were presented for solving a three-level decision maker's model. This approach used stochastic parameters in the objective function to solve a three level quadratic programming problem. The probabilistic nature of the objective functions were converted to an equivalence deterministic one and then a fuzzy programming technique used in each level to generate Pareto optimal solution for these problems.

Competing Interests

Authors have declared that no competing interests exist.

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