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Dynamic Response of Non-Uniform Elastic Structure Resting on Exponentially Decaying Vlasov Foundation under Repeated Rolling Concentrated Loads

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Authors' contributions

This work was carried out in collaboration between both authors. Author JSA designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript. Author AOF managed the analyses of the study. Author JSA managed the literature searches. Both authors read and approved the final manuscript.

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Abstract

In this paper, the motion of a non-uniform elastic structure resting on exponentially decaying Vlasov foundation and under repeated rolling concentrated loads moving with constant velocity is analyzed. The governing equation is a fourth order partial differential equation. The solution technique is based on the method of Galerkin with series representation of Heaviside function. The result shows that, an increase in the values of the structural parameters such as foundation stiffness, foundation modulus and distance x along span of the beam reduces the response amplitude of the beam for the dynamic problem.

Keywords: Non-uniform elastic structure; repeated rolling concentrated loads; exponentially decaying Vlasov foundation; Galerkin's method.

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1 Introduction

Studies in structural dynamics dealing with moving loads on elastic structures are enormous and have been enriched in the last few decades by the development of high speed railway networks, elevated road-ways, highway bridges especially cable-stayed and suspension bridges etc., in the developed and developing countries. The fundamental problems have been the increasing high speeds and weights of the vehicles which this structure carries. In particular, the dynamic response of a simply supported beam, traversed by a constant force moving at a uniform speed was first studied by Krylov [1]. His results were obtained using the method of Eigen-functions. He assumed that the mass of the load is smaller than that of the beam. Later, Timoshenko [2] used energy methods to obtain solution in series form for simply supported finite beam on an elastic foundation subjected to time dependent point loads moving with uniform velocities across the beam. Kenny [3] similarly investigated the dynamic response of infinite beams on an elastic foundation under the influence of loads moving at constant speeds. He included the effects of viscous damping in the governing differential equation of motion. Steel [4] also investigated the response of a finite simply supported Bernoulli-Euler beam to a unit force moving at a uniform velocity. He analyzed the effects of this moving force on beams with and without an elastic foundation using a considerably simpler vector formulation with a Laplace rather than Fourier transformation. In a much later development Oni, [5] considered the problem of a harmonic time-variable concentrated force moving at a uniform velocity over a finite deep beam. The methods of integral transformations were used. Series solution which converges was obtained for the deflection of the simply supported beams and analyzed for various speeds of loads. Just as for elastic beams, the problem of dynamic response of an elastic plate to moving loads when the mass effect of the moving load is neglected has been tackled by many authors. The study of non-uniform elastic structure resting on exponentially decaying Vlasov foundation (i.e. a bi-parametric foundation model that shows the relationship between the foundation reaction and vertical deflection by incorporating shear modulus into the Winkler foundation model) and under repeated rolling concentrated loads moving with constant velocity forms a very important structural element in engineering design and construction. It has also become the objective of various researchers in the field of Applied Mathematics. In general, problems of this type are mathematically complex if analytical approach is used. Thus, most of the research works available in the Literature are those in which Numerical technique is used. This is due to great amount of computational labour which is required both to set up and solve the necessary equations. A major break-through in this field of research is the work of Timoshenko [2] who gave impetus to research work in this area of study. The analysis of the dynamic response of a simple beam continuously supported by a viscoelastic foundation to a moving load, moving at variable speed was considered. The analysis reveals several resonance conditions depending on the viscoelasticity of the foundation. Also a theory for the response to an arbitrary number of concentrated moving masses of a rectangular plate continuously supported by an elastic Pasternak type foundation was developed [6]. It was found that the critical speeds of the system increased with increase in the values of the foundation moduli whether the inertia of the moving load is considered or not. The displacement response of a simply supported non-uniform beam resting on an elastic foundation to several moving load was later taken up and concluded that the maximum transverse deflection of the beam is always greater than the displacement of the moving mass [7]. A modification of the asymptotic method was used to simplify the resulting sequence of differential equation [8]. Sayad et al. Study vibration analyses of a tapered beam with exponentially varying thickness resting on Winkler foundation using the differential transform method [9]. It is well known that in the dynamical system like this, analytical methods are desirable as a method of solution, as there often shed light on vital information about the vibrating system.

Thus, this paper studied the dynamic response of non-uniform elastic structure resting on exponentially decaying Vlasov foundation and under repeated rolling concentrated loads using analytical approach. Several numerical examples will also be presented. It is assumed that the speed at which the load traverses the structural element is constant.

2 Mathematical Model

The motion of a non-uniform elastic structure resting on exponentially decaying Vlasov foundation and under repeated rolling concentrated loads moving with constant velocity shown in figure (1) is governed by the following fourth order partial differential equation

$$\frac{\partial^2}{\partial x^2}[F_m(x,t)] + \mu(x)\frac{\partial^2 V}{\partial t^2} - N\frac{\partial^2 V(x,t)}{\partial x^2} + d(x)\frac{\partial V(x,t)}{\partial t} + K(x)V(x,t) - G(x)\frac{\partial^2 V(x,t)}{\partial x^2} = f(x,t)\,Z \qquad (1)$$

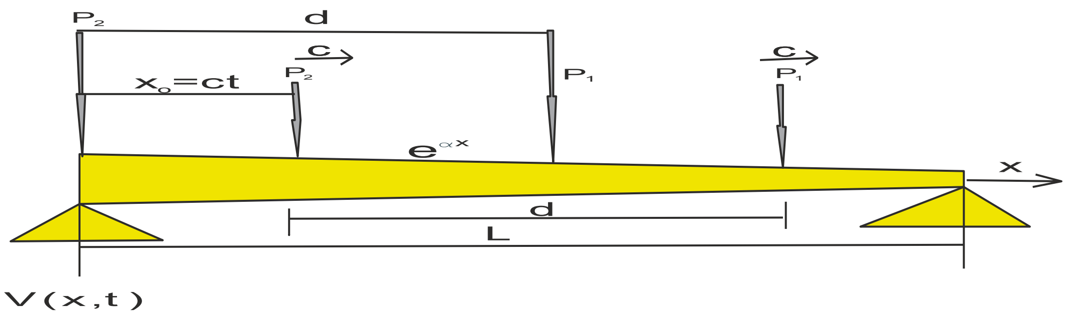

Fig. 1. Geometry of a non-uniform beam on exponentially Vlasov foundation

Where *μ(x)* is the variable mass per unit length *L* of the beam, *t* is the time, *N* is the constant axial force, *d(x)* is the variable material damping intensity, $F_m(x, t)$ is the flexural moment acting on the beam across section, *V(x, t)* is the beam lateral displacement, *f(x, t)* is the rolling concentrated loads. *K(x)* and *G(x)* are the variable foundation stiffness and foundation modulus respectively.

The flexural moment acting on the beam across section is related to the vertical response as

$$F_m(x,t) = EI(x)\frac{\partial^2}{\partial x^2}V(x,t) \qquad (2)$$

Where *EI(x)* is the flexural rigidity of the beam and *E* is the Young's modulus.

The non-uniform characteristics distribution of the beam may be assumed as power functions Mohamed [10]. The parameters β and n are used to approximate the actual non-uniformity of the beam given as

$$\mathrm{I(x)} = I_o(1+\beta x)^{n+2}; \quad \mu(x) = \mu_o(1+\beta x)^n; \quad \mathrm{d(x)} = d_o(1+\beta x)^n \qquad (3)$$

Where *I(x)* is the variable moment of inertia of the beam, I_o, μ_o and d_o are the beam characteristic constants at x = 0.

In this paper, we adopt the example in Omolofe [11] to define exponentially decaying Vlasov foundation as

$$\mathrm{K(x)} = K_o e^{\lambda_1 x} \qquad \mathrm{G(x)} = G_o e^{\lambda_2 x} \qquad (4)$$

Where λ_1 and λ_2 are constants, K_o and G_o are the elastic foundation constants.

2.1 The boundary conditions

The boundary conditions depend on the constraints at the beam ends. However, for a simply supported beam whose length is L, the vertical displacement at the beam ends are given as

$$V(0,t) = V(L,t) = 0; \quad V''(0,t) = V''(L,t) = 0 \tag{5}$$

It is assumed that the initial conditions without any loss of generality is given by

$$V(x,0) = 0 = \ddot{V}(x,0) \tag{6}$$

Where dash and dot are the derivatives with respect to x and t respectively.

Furthermore, we adopt the example in Shahin (2010) to define the rolling concentrated Loads *f(x, t)* as

$$f(x,t) = P_2\delta \ \ (x - ct) + P_1\delta[x - (ct + d)] \tag{7}$$

where P_i is the constant magnitude of moving concentrated load, $i = 1,2$, d is the distance between the repeated moving loads, $\delta(x - ct)$ represent the Dirac delta function, c is the velocity of the i^{th} particle of the system. Substituting Eqns. (2), (3), (4) and (7) into (1), taking n=1, one obtains

$$EI_o\left[(1+\beta x)^3\frac{\partial^4 V(x,t)}{\partial x^4} + 2\frac{\partial}{\partial x}(1+\beta x)^3\frac{\partial^3 V(x,t)}{\partial x^3} + \frac{\partial^2}{\partial x^2}(1+\beta x)^3\frac{\partial^2 V(x,t)}{\partial x^2}\right] + \mu_o(1+\beta x)\frac{\partial^2 V}{\partial t^2} - N\frac{\partial^2 V(x,t)}{\partial x^2} + d_o(1+\beta x)\frac{\partial V(x,t)}{\partial t} + K_o e^{\lambda_1 x}V(x,t) - G_o e^{\lambda_2 x}\frac{\partial^2 V(x,t)}{\partial x^2} = P_2\delta(x-ct) + P_1\delta[x-(ct+d)] \tag{8}$$

2.2 Approximate analytical solution

2.2.1 Analysis of non-uniform beam resting on exponentially decaying Vlasov foundation when the distance between the rolling concentrated loads is less than the length of the beam

In this section, in order to compute the response displacement of the beam due to moving repeated rolling concentrated loads, we shall use an elegant technique called Galerkin's method often used in solving diverse problems involving mechanical vibrations Ojih (2013). This method requires that the solution of the deflection of the coupled equation is expressed as

$$V(x,t) = \sum_{j=1}^{N} Y_j(t)U_j(x) \tag{9}$$

Where $Y_j(t)$ coordinates in modal are space and $U_j(x)$ are the normal modes of free vibration written as

$$U_j(x) = sin\lambda_j x + A_j cos\lambda_j x + B_j sinh\lambda_j x + C_j cosh\lambda_j x \tag{10}$$

Where the constants A_j, B_j & C_j define the space and amplitude of the beam vibration. Their values depend on the boundary condition associated with the structure. Thus, for a simply supported beam, using eq. (5), it can be shown that

$$A_j + C_j = 0; \qquad -A_j + C_j = 0; \tag{11}$$

$$sin\lambda_j + A_j\,cos\lambda_j + B_j\,sinh\lambda_j + C_j\,cosh\lambda_j = 0; \quad -sin\lambda_j - A_j\,cos\lambda_j + B_j\,sinh\lambda_j + C_j\,cosh\lambda_j = 0 \tag{12}$$

Solving the equations in (11) and (12) simultaneously, it is easy to show that

$$A_j = B_j = C_j = 0 \ and \ \lambda_j = \frac{j\pi}{L} \tag{13}$$

Thus, for a beam with simple support at both ends, Eqn. (10) becomes

$$U_j(x) = sin\frac{j\pi x}{L} \quad (14)$$

Thus, in view of (9) and (12), after some simplification and rearrangement, Eqn. (8) becomes

$$\sum_{j=1}^{N}\left(EI_o\left[\left(\frac{j\pi}{L}\right)^4 D_1(x)sin\frac{j\pi x}{L} - 6\beta\left(\frac{j\pi}{L}\right)^3 D_2(x)cos\frac{j\pi x}{L} - 6\beta^2\left(\frac{j\pi}{L}\right)^2 D_3(x)sin\frac{j\pi x}{L}\right] + N\left(\frac{j\pi}{L}\right)^2 sin\frac{j\pi x}{L} + K_o e^{\lambda_1 x} sin\frac{j\pi x}{L} + G_o\left(\frac{j\pi}{L}\right)^2 e^{\lambda_2 x} sin\frac{j\pi x}{L}\right)Y_j(t) + \left[d_o(1+\beta x)sin\frac{j\pi x}{L}\right]\dot{Y}_j(t) + \left[\mu_o(1+\beta x)sin\frac{j\pi x}{L}\right]\ddot{Y}_j(t) - P_2\delta(x-ct) + P_1\delta[x-(ct+d)] = 0 \quad (15)$$

To determine $Y_j(t)$, the expressions on the left hand side of (15) are required to be orthogonal to the function $sin\frac{m\pi x}{L}$. Thus, equation (15) becomes

$$r_1(j,m)\ddot{Y}_j(t) + r_2(j,m)\dot{Y}_j(t) + r_3(j,m)Y_j(t) = P_2I_7 + P_1I_8 \quad (16)$$

Where

$$r_1(j,m) = \mu_o I_3 \qquad r_2(j,m) = d_o I_3$$

$$r_3(j,m) = EI_o(S_1 - S_2 - S_3) + N\left(\frac{j\pi}{L}\right)^2 I_4 + K_o I_5 + G_o\left(\frac{j\pi}{L}\right)^2 I_6$$

$$S_1 = \left(\frac{j\pi}{L}\right)^4 I_1 \qquad S_2 = 6\beta\left(\frac{j\pi}{L}\right)^3 I_2 \qquad S_3 = 6\beta^2\left(\frac{j\pi}{L}\right)^2 I_3$$

$$I_1 = \int_0^L D_1(x)sin\frac{j\pi x}{L}\ sin\frac{m\pi x}{L}\ dx \qquad I_2 = \int_0^L D_2(x)cos\frac{j\pi x}{L}\ sin\frac{m\pi x}{L}\ dx$$

$$I_3 = \int_0^L D_3(x)sin\frac{j\pi x}{L}\ sin\frac{m\pi x}{L}\ dx \qquad I_4 = \int_0^L sin\frac{j\pi x}{L}\ sin\frac{m\pi x}{L}\ dx$$

$$I_5 = \int_0^L e^{\lambda_1 x}sin\frac{j\pi x}{L}\ sin\frac{m\pi x}{L}\ dx \qquad I_6 = \int_0^L e^{\lambda_2 x}sin\frac{j\pi x}{L}\ sin\frac{m\pi x}{L}\ dx$$

$$I_7 = \int_0^L \delta(x-ct)\,sin\frac{m\pi x}{L}\ dx \qquad I_8 = \int_0^L \delta[x-(ct+d)]\,sin\frac{m\pi x}{L}\ dx$$

$$D_1(x) = (1+\beta x)^3 \qquad D_2(x) = (1+\beta x)^2 \qquad D_2(x) = (1+\beta x) \quad (17)$$

Further simplification and rearrangement of (16) after substituting the values of the integrals' results in (17) into (16), one obtains

$$\ddot{Y}_j(t) + Q_1\dot{Y}_j(t) + Q_2Y_j(t) = Q_3 sin\emptyset t + Q_4\sin(\emptyset t + \theta) \quad (18)$$

Where

$$Q_1 = \frac{r_2(j,m)}{r_1(j,m)};\quad Q_2 = \frac{r_3(j,m)}{r_1(j,m)};\quad Q_3 = \frac{P_2}{r_1(j,m)};\quad Q_4 = \frac{P_1}{r_1(j,m)};\quad \emptyset = \frac{m\pi c}{L};\quad \theta = \frac{m\pi d}{L} \quad (19)$$

Equation (18) is a second order ordinary differential equation, therefore, subjecting the equation to a Laplace transform defined as

$$\eta = \int_0^{\infty}(\sim)e^{-st}dt \tag{20}$$

In conjunction with the initial conditions defined in (6), gives the following simple algebraic equation

$$Y_j(s) = \frac{1}{\varphi_2 - \varphi_1}\left[(Q_3 + H_{4a})\left(\frac{\emptyset}{s^2 + \emptyset^2}\right) + H_{4b}\left(\frac{s}{s^2 + \emptyset^2}\right)\right]\left(\frac{1}{s + \varphi_1} - \frac{1}{s + \varphi_2}\right) \tag{21}$$

Where

$$\varphi_1 = \tfrac{1}{2}\left(Q_1 - \sqrt{{Q_1}^2 - 4Q_2}\right) \quad \varphi_2 = \tfrac{1}{2}\left(Q_1 + \sqrt{{Q_1}^2 - 4Q_2}\right) \quad H_{4a} = Q_4 cos\theta \quad H_{4b} = Q_4 sin\theta \tag{22}$$

Thus, the equation reduces to that of finding Laplace inversion of (21), so that the Laplace inversion of $Y_j(s)$ is the convolution of (21) defined as

$$f(s) * g(s) = \int_0^t f(t-u)g(u)du \tag{23}$$

Where

$$f(s) = \left(\frac{1}{s + \varphi_1} - \frac{1}{s + \varphi_2}\right) \quad and \quad g(s) = \left[(Q_3 + H_{4a})\left(\frac{\emptyset}{s^2 + \emptyset^2}\right) + H_{4b}\left(\frac{s}{s^2 + \emptyset^2}\right)\right] \tag{24}$$

Thus, the Laplace inversion of (21) is given as

$$Y_j(t) = \frac{1}{\varphi_2 - \varphi_1}[P_{3a}(I_{15} - I_{16}) + H_{4b}(I_{17} - I_{18})] \tag{25}$$

Where

$$I_{15} = \int_0^t e^{\varphi_1(t-u)} sin\emptyset u\, du \qquad I_{16} = \int_0^t e^{\varphi_2(t-u)} sin\emptyset u\, du \qquad I_{17} = \int_0^t e^{\varphi_1(t-u)} cos\emptyset u\, du$$
$$I_{18} = \int_0^t e^{\varphi_2(t-u)} cos\emptyset u\, du \tag{26}$$

It is easy to show that

$$I_{15} = \frac{1}{\emptyset^2 + {\varphi_1}^2}(\emptyset e^{\varphi_1 t} - \varphi_1 sin\emptyset t - \emptyset sin\emptyset t) \qquad I_{16} = \frac{1}{\emptyset^2 + {\varphi_2}^2}(\emptyset e^{\varphi_2 t} - \varphi_2 sin\emptyset t - \emptyset sin\emptyset t)$$
$$I_{17} = \frac{1}{\emptyset^2 + {\varphi_1}^2}(\varphi_1 e^{\varphi_1 t} - \varphi_1 cos\emptyset t + \emptyset sin\emptyset t) \qquad I_{18} = \frac{1}{\emptyset^2 + {\varphi_2}^2}(\varphi_2 e^{\varphi_2 t} - \varphi_2 cos\emptyset t + \emptyset sin\emptyset t) \tag{27}$$

Substituting (27) into (25), after some rearrangement, one obtains

$$Y_j(t) = \frac{1}{(\varphi_2 - \varphi_1)(\emptyset^2 + {\varphi_1}^2)(\emptyset^2 + {\varphi_2}^2)}\{P_{3a}[(\emptyset^2 + {\varphi_2}^2)(\emptyset e^{\varphi_1 t} - \varphi_1 sin\emptyset t - \emptyset sin\emptyset t) - (\emptyset^2 + {\varphi_1}^2)(\emptyset e^{\varphi_2 t} - \varphi_2 sin\emptyset t - \emptyset sin\emptyset t)] + H_{4b}[(\emptyset^2 + {\varphi_2}^2)(\varphi_1 e^{\varphi_1 t} - \varphi_1 cos\emptyset t + \emptyset sin\emptyset t) - (\emptyset^2 + {\varphi_1}^2)(\varphi_2 e^{\varphi_2 t} - \varphi_2 cos\emptyset t + \emptyset sin\emptyset t)]\} \tag{28}$$

In view of (9), (19) and (22), we have

$$V(x,t) = \sum_{j=1}^{N} \frac{P_1}{(\emptyset^2+\varphi_1{}^2)(\emptyset^2+\varphi_2{}^2)(\varphi_2-\varphi_1)r_1(j,m)} \Big\{\Big(\frac{P_2}{P_1} + cos\theta\Big)[(\emptyset^2+\varphi_2{}^2)(\emptyset e^{\varphi_1 t} - \varphi_1 sin\emptyset t - \emptyset sin\emptyset t) - (\emptyset^2+\varphi_1{}^2)(\emptyset e^{\varphi_2 t} - \varphi_2 sin\emptyset t - \emptyset sin\emptyset t)] + sin\theta\,[(\emptyset^2+\varphi_2{}^2)(\varphi_1 e^{\varphi_1 t} - \varphi_1 cos\emptyset t + \emptyset sin\emptyset t) - (\emptyset^2+\varphi_1{}^2)(\varphi_2 e^{\varphi_2 t} - \varphi_2 cos\emptyset t + \emptyset sin\emptyset t)]\Big\} \times \frac{\sin j\pi x}{L} \tag{29}$$

Eqn. (29) is the transverse displacement of the non-uniform elastic structure resting on exponentially decaying Vlasov foundation under repeated rolling concentrated loads when the distance between P_1 and P_2 is less than the length L of the beam.

2.2.2 Analysis of non-uniform beam resting on exponentially decaying Vlasov foundation when the distance between the rolling concentrated loads is equal to the length of the beam

In this section, we consider the case when the distance between the concentrated loads is equal to the length of the beam. This implies that one load is leaving the beam as the other begins to travel onto the beam at the left end, and if the time t is remeasured from this instant, the differential equation governing the transverse motion of the beam may be written in the form

$$EI_o\Big[(1+\beta x)^3\frac{\partial^4 V(x,t)}{\partial x^4} + 2\frac{\partial}{\partial x}(1+\beta x)^3\frac{\partial^3 V(x,t)}{\partial x^3} + \frac{\partial^2}{\partial x^2}(1+\beta x)^3\frac{\partial^2 V(x,t)}{\partial x^2}\Big] + \mu_o(1+\beta x)\frac{\partial^2 V}{\partial t^2} - N\frac{\partial^2 V(x,t)}{\partial x^2} + d_o(1+\beta x)\frac{\partial V(x,t)}{\partial t} + K_o e^{\lambda_1 x}V(x,t) - G_o e^{\lambda_2 x}\frac{\partial^2 V(x,t)}{\partial x^2} = P_2\delta(x-ct) \tag{30}$$

Like in the previous section, closed form solution to (30) is sought using (9). Following the same arguments as in the previous section after some simplification and rearrangements, equation (30) becomes

$$r_1(j,m)\ddot{Y}_j(t) + r_2(j,m)\dot{Y}_j(t) + r_3(j,m)Y_j(t) = P_2 sin\frac{m\pi ct}{L} \tag{31}$$

Equation (31) is analogous to (16), thus subjecting (31) to a Laplace transform in conjunction with the boundary conditions stated in (6) and using convolution theory, after inversion, one obtains

$$V(x,t) = \sum_{j=1}^{N} \frac{P_2}{(\emptyset^2+\varphi_1{}^2)(\emptyset^2+\varphi_2{}^2)(\varphi_2-\varphi_1)r_1(j,m)}\{[(\emptyset^2+\varphi_2{}^2)(\emptyset e^{\varphi_1 t} - \varphi_1 sin\emptyset t - \emptyset sin\emptyset t) - (\emptyset^2+\varphi_1{}^2)(\emptyset e^{\varphi_2 t} - \varphi_2 sin\emptyset t - \emptyset sin\emptyset t)]\} \times \frac{\sin j\pi x}{L} \tag{32}$$

Eqn. (32) is the transverse displacement of the non-uniform elastic structure resting on exponentially decaying Vlasov foundation under repeated rolling concentrated loads when the distance between P_1 and P_2 is equal to the length L of the beam.

3 Results and Discussion

The transversed displacement of a non-uniform elastic beam may increase without bound. Thus one is interested in resonance condition. Eqs (29) and (32) clearly depicts that the non-uniform beam resting on exponentially decaying Vlasov foundation will grow without bound whenever

$$\varphi_1 = \varphi_2\,; \qquad \varphi_1{}^2 = -\,\emptyset^2\,; \qquad \varphi_2{}^2 = -\,\emptyset^2 \tag{33}$$

and the velocity at which this occurs is known as critical velocity which is given as

$$c = \frac{L}{2m\pi}\left[4Q_2 + 2Q_1\sqrt{{Q_1}^2 - 4Q_2} - 2{Q_1}^2\right]^{\frac{1}{2}} \quad or \quad c = \frac{L}{2m\pi}\left[4Q_2 - 2Q_1\sqrt{{Q_1}^2 - 4Q_2} - 2{Q_1}^2\right]^{\frac{1}{2}} \tag{34}$$

For the purpose of numerical analysis of the forgoing problem, the velocity of the moving load and the length of non-uniform elastic beam are 30m/s and 15m respectively. Furthermore, $\mathrm{E} = 2.02\mathrm{e}+11$, $\beta = 0.025, I_o = 0.0012\ \lambda_1 = 0.5,\ \lambda_2 = 0.4,\ d_o = 0.05,\ d = 1.4,\ L = 3,\ c = 50$. The deflection profile of a non-uniform elastic structure resting on exponentially decaying Vlasov foundation under repeated rolling concentrated loads when the distance between P_1 and P_2 is less than the length L of the beam is shown in Fig (2a) – Fig (2f). It is observed that as the values of the varying parameters are increased, the response amplitudes decreased. The same effect is shown when the distance between P_1 and P_2 is equal to the length L of the beam in Fig. 3.

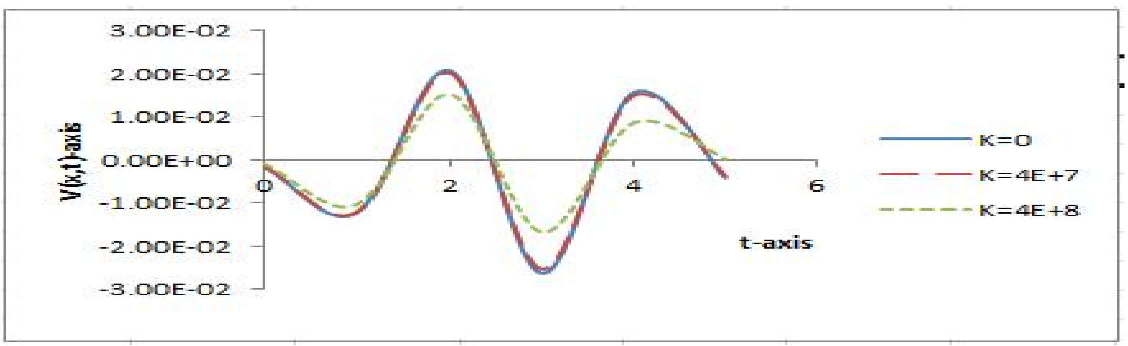

Fig. 2(a). Transverse displacement of the non-uniform elastic structure resting on exponentially decaying Vlasov foundation under repeated rolling concentrated loads when the distance between P_1 and P_2 is equal to the length L of the beam for various value of foundation stiffness K and for fixed values of N (40000), P1(16000), P2(18000) , d(1.4) and G(40000)

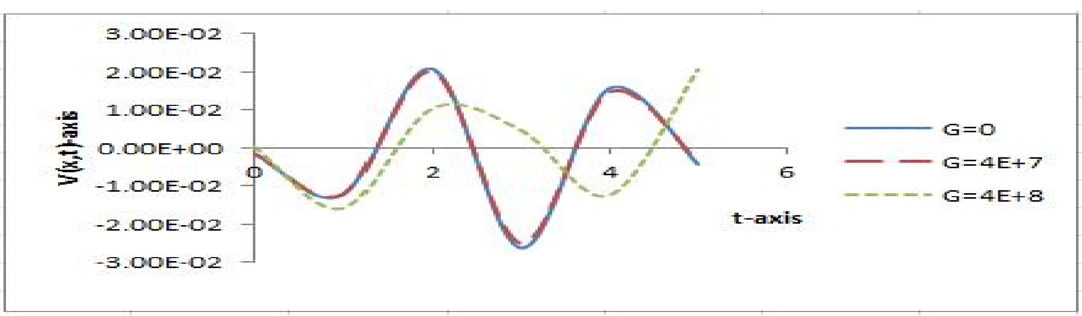

Fig. 2(b). Transverse displacement of the non-uniform elastic structure resting on exponentially decaying Vlasov foundation under repeated rolling concentrated loads when the distance between P_1 and P_2 is equal to the length L of the beam for various value of foundation modulus G and for fixed values of N (40000), P1(16000), P2(18000), d(1.4) and K(40000)

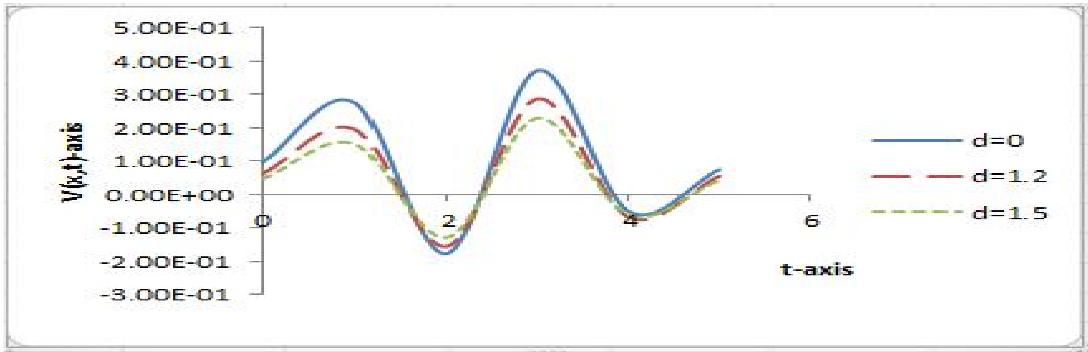

Fig. 2(c). Transverse displacement of the non-uniform elastic structure resting on exponentially decaying Vlasov foundation under repeated rolling concentrated loads when the distance between P_1 and P_2 is equal to the length L of the beam for various value of distance d and for fixed values of N (40000), P1(16000), P2(18000), K(40000) and G (40000)

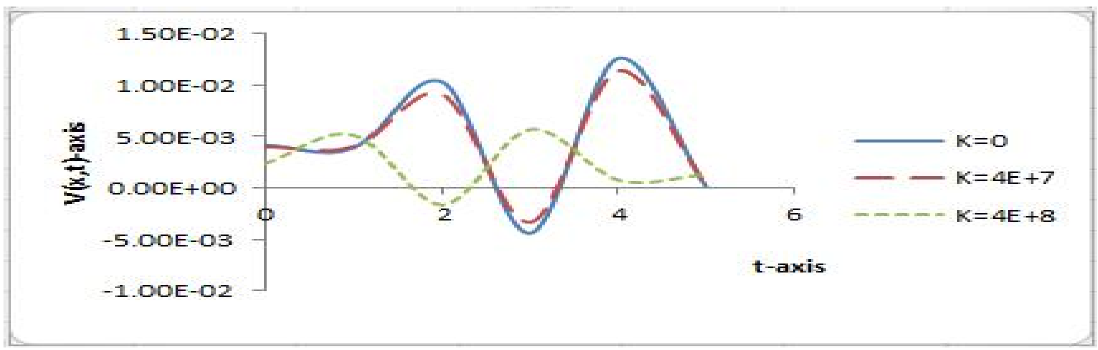

Fig. 2(d). Transverse displacement of the non-uniform elastic structure resting on exponentially decaying Vlasov foundation under repeated rolling concentrated loads when the distance between P_1 and P_2 is equal to the length L of the beam for various value of foundation stiffness K and for fixed values of N (40000), P1(18000), P2(18000) , d(1.4) and G(40000)

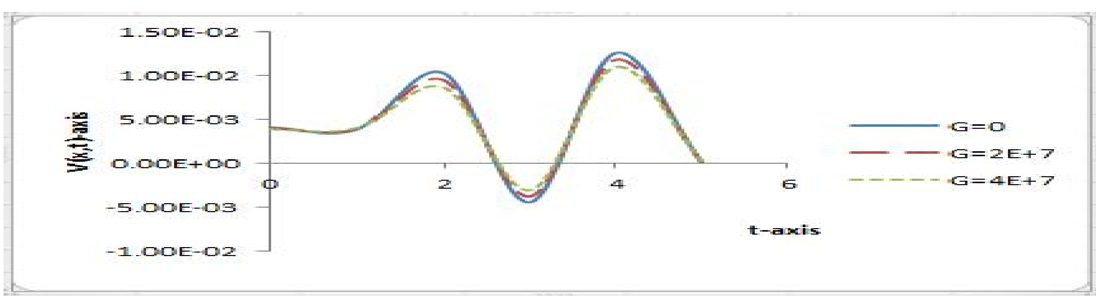

Fig. 2(e). Transverse displacement of the non-uniform elastic structure resting on exponentially decaying Vlasov foundation under repeated rolling concentrated loads when the distance between P_1 and P_2 is equal to the length L of the beam for various value of foundation modulus G and for fixed values of N (40000), P1(18000), P2(18000), d(1.4) and K(40000)

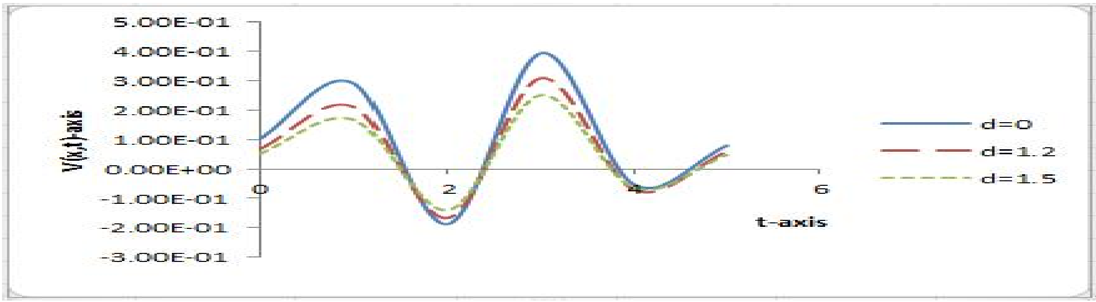

Fig. 2(f). Transverse displacement of the non-uniform elastic structure resting on exponentially decaying Vlasov foundation under repeated rolling concentrated loads when the distance between P_1 and P_2 is equal to the length L of the beam for various value of distance d and for fixed values of N (40000), P1(18000), P2(18000), K(40000) and G (40000)

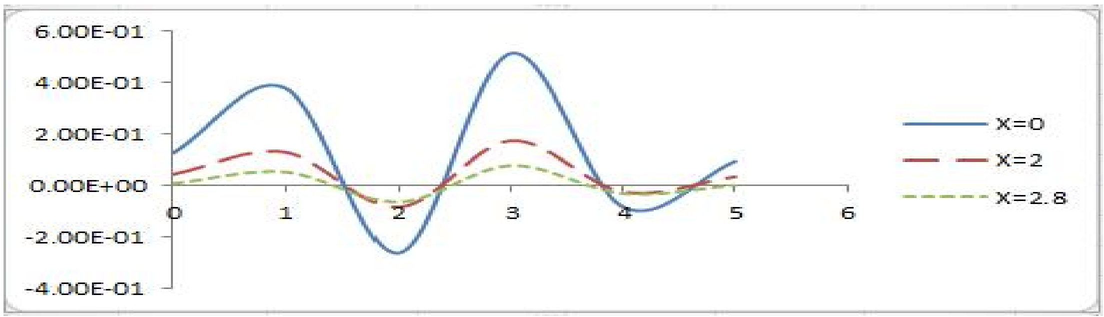

Fig. 3. Transverse displacement of the non-uniform elastic structure resting on exponentially decaying Vlasov foundation under repeated rolling concentrated loads when the distance between P_1 and P_2 is equal to the length L of the beam for various value of distance d and for fixed values of N (40000), P1(18000), P2(18000), K(40000) and G (40000)

4 Conclusion

An analytical solution is presented for the deflection response of non-uniform elastic structure resting on exponentially decaying Vlasov foundation and under repeated rolling concentrated loads moving with constant velocity. The solution technique is based on Galerkin's method and Laplace transformation. The analysis exhibited the following features:

- The critical speeds of the system increase with an increase in the values of foundation stiffness, foundation modulus and exponential factor in the problem of non-uniform elastic structure resting on exponentially decaying Vlasov foundation and under repeated rolling concentrated loads moving with constant velocity.
- As the foundation parameters increased, the transverse deflection of the beam model decreased.

Thus, the risk of resonance was reduced as the value of the foundation parameters increased

Competing Interests

Authors have declared that no competing interests exist.

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