



The space $M_4(\Gamma_0(100), (5/.)$) and Representation Numbers of Some Octonary Quadratic Forms

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

The determination of the number of representations of a positive integer by certain octonary quadratic forms are given in the literature. For instance, the formulas $N(1^{2a}, 2^{2b}, 3^{2c}, 6^{2d}; n)$ for the **nine octonary** quadratic forms are given with $a = 1, b = 2, c = 3$ and $d = 6$. Moreover, the formulas for $N(1^a, 3^b, 9^c; n)$ for several **octonary** quadratic forms have been given by Alaca. Here, by using MAGMA, we determine the formulas, for $N(1^a, 5^b, 25^c; n)$ by Eisenstein series and eta quotients for several octonary quadratic forms whose theta functions are in $M_4(\Gamma_0(100), \chi)$.

Keywords: Octonary quadratic forms; representations; theta functions; Dedekind eta function; Eisenstein series; modular forms; Dirichlet character.

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1 Introduction

It is interesting and important to determine explicit formulas of the representation number of positive definite quadratic forms.

The work on representation number $\#\{(x_1, x_2) \in \mathbb{Z}^2 | n = x_1^2 + x_2^2\}$ of quadratic form $x^2 + y^2$ has been started by Fermat in 1640. It would be helpful to obtain such simple formulas for other positive definite quadratic forms Q so that the author would be able to understand the number of solutions of the equation $Q = n$ for any positive integer n .

Later the formula,

$$\#\{(x_1, x_2) \in \mathbb{Z}^2 | n = x_1^2 + x_2^2\} = 4 \left(\sum_{d|n, d \text{ is odd}} (-1)^{\frac{d-1}{2}} \right)$$

has been proved by Euler. First systematic treatment of binary quadratic forms is due to Legendre. Afterwards it was advanced by Jacobi, with the proof of

$$\#\{(x_1, \dots, x_4) \in \mathbb{Z}^4 | n = x_1^2 + x_2^2 + x_3^2 + x_4^2\} = 8 \left(\sum_{d|n, 4 \nmid d} d \right).$$

The theory was advanced much further by Gauss in *Disquisitiones Arithmetica*. The research of Gauss strongly influenced both the arithmetical theory of quadratic forms in more than two variables and subsequent development of algebraic number theory. Since then, there are many more representation number formulas obtained for quadratic forms. Especially, by means of the deep theorems of Hecke [1] and Schoeneberg [2], modular forms have been used in the representation number of several quadratic forms.

The generalized theta series ([3],[4]), quasimodular forms ([5],[6],[7]) and several other methods have been also used for the representation number formulas.

For $a_1, \dots, a_8 \in \mathbb{N}$ and a nonnegative integer n , defines

$$N(a_1, \dots, a_8; n) := \text{card}\{(x_1, \dots, x_8) \in \mathbb{Z}^8 | n = a_1 x_1^2 + \dots + a_8 x_8^2\}.$$

Clearly $N(a_1, \dots, a_8; 0) = 1$ and, without loss of generality can assume that,

$$a_1 \leq \dots \leq a_8, \gcd\{a_1, \dots, a_8\} = 1$$

The formulas for $N(1^i, 3^j, 9^k; n)$ for several **octonary** quadratic forms have been given by Alaca [8]. Here, determines formulae, for $N(1^i, 5^j, 25^k; n)$ for several octonary quadratic forms.

The divisor function $\sigma_i(n)$ is defined for a positive integer i by

$$\sigma_i(n) := \begin{cases} \sum_{d \text{ positive integer}, d|n} d^i, & \text{if } n \text{ is a positive integer} \\ 0 & \text{if } n \text{ is not a positive integer.} \end{cases} \quad (1)$$

The Dedekind eta function and the theta function are defined by

$$\eta(z) := q^{1/24} \prod_{n=1}^{\infty} (1 - q^n), \varphi(q) := \sum_{n \in \mathbb{Z}} q^{n^2}, \quad (2)$$

where,

$$q := e^{2\pi iz}, z \in H = \{x + iy : y > 0\} \quad (3)$$

and, an eta quotient of level N is defined by

$$f(z) := \prod_{m|N} \eta(mz)^{a_m}, N, m \in \mathbb{N}, a_m \in \mathbb{Z}. \quad (4)$$

Here, the author gives the following Lemma, see[[9] Theorem 1.64] about the modularity of an eta quotient.

Lemma 1: An eta quotient of level N is a meromorphic modular form of weight $k = \frac{1}{2} \sum_{m|N} a_m$ on $\Gamma_0(N)$, with character χ , having rational coefficients with respect to q if

$$\begin{aligned} a) \sum_{m|N} a_m &\text{ is even,} \\ b) \sum_{m|N} ma_m &\equiv \sum_{m|N} \frac{N}{m} a_m \equiv 0 \pmod{24}, \\ c) \chi(d) &= \left(\frac{(-1)^k \prod_{m|N} m^{a_m}}{d} \right), d \in \mathbb{N}. \end{aligned}$$

Now, let's consider octonary quadratic forms of the form

$$Q := x_1^2 + \cdots + x_a^2 + 5(x_{a+1}^2 + \cdots + x_{a+b}^2) + 25(x_{a+b+1}^2 + \cdots + x_{a+b+c}^2),$$

where $a, b, c \in \mathbb{Z}$, $0 < a \leq 8$, $0 \leq b \leq 8$, $b \equiv 1 \pmod{2}$, $0 \leq c \leq 8$, and $a + b + c = 8$. We list those (a, b, c) in Table 1.

We write $N(1^a, 5^b, 25^c; n)$ to denote the number of representations of n by an octonary quadratic form (a, b, c) . Its theta function is obviously

Table1.

a	b	c	a	b	c	a	b	c	a	b	c
1	1	6	1	5	2	2	1	5	2	5	1
1	3	4	1	7	0	2	3	3	3	1	4
3	3	2	4	1	3	5	1	2	6	1	1
3	5	0	4	3	1	5	3	0	7	1	0

$$\Theta_Q = \varphi^a(q) \varphi^b(q^5) \varphi^c(q^{25}).$$

Formulas for $N(1^{2i}, 2^{2j}, 3^{2k}, 6^{2l}; n)$ for the nine octonary quadratic forms $(2i, 2j, 2k, 2l) = (8, 0, 0, 0), (2, 6, 0, 0), (4, 4, 0, 0), (6, 2, 0, 0), (2, 0, 6, 0), (4, 0, 4, 0), (6, 0, 2, 0), (4, 0, 0, 4),$ and $(0, 4, 4, 0)$ appear in the literature, [10],[11],[12],[13],[14]. Moreover, the formulas for $N(1^i, 3^j, 9^k; n)$ for twenty octonary quadratic forms have been given by Alaca. Here, the author determines formulae, for $N(1^i, 5^j, 25^k; n)$ for sixteen octonary quadratic forms.

Here, the author will classify all triples (a, b, c) for which Θ_Q is a modular form of level 100 and weight 4 with nebentypus χ , where χ is the Dirichlet character mod 100 determined by the Kronecker Symbol $(\frac{5}{n})$. Then we will obtain their representation numbers in terms of the coefficients of Eisenstein series and some eta quotients.

First, by the following Theorem, we characterize the facts that

$$\varphi^a(q) \varphi^b(q^5) \varphi^c(q^{25})$$

are in $M_4(\Gamma_0(100), \chi)$.

Theorem 2 Let

$$Q := x_1^2 + \cdots + x_a^2 + 5(x_{a+1}^2 + \cdots + x_{a+b}^2) + 25(x_{a+b+1}^2 + \cdots + x_{a+b+c}^2)$$

where, $a, b, c \in \mathbb{Z}$, $0 < a \leq 8$, $0 \leq b \leq 8$, $0 \leq c \leq 8$ and $a + b + c = 8$, be an octonary quadratic form. Then its theta series is of the form

$$\begin{aligned} \Theta_Q &= \varphi^a(q) \varphi^b(q^5) \varphi^c(q^{25}) = \\ &\eta^{-2a}(q) \eta^{5a}(q^2) \eta^{-2a}(q^4) \eta^{-2b}(q^5) \eta^{5b}(q^{10}) \eta^{-2b}(q^{20}) \eta^{-2c}(q^{25}) \eta^{5c}(q^{50}) \eta^{-2c}(q^{100}) \end{aligned}$$

Moreover, it is in $M_4(\Gamma_0(100), \chi)$ if and only if $b \equiv 1 \pmod{2}$, i.e., (a, b, c) is given in the Table 1.

Proof. It follows from the Lemma 1, holomorphicity criterion in [[15] Corollary 2.3,p.37] and the

fact that

$$\varphi(q) = \frac{\eta^5(q^2)}{\eta^2(q) \eta^2(q^4)}.$$

The condition

$$1^{-2a} 2^{5a} 4^{-2a} 5^{-2b} 10^{5b} 20^{-2b} 25^{-2c} 50^{5c} 100^{-2c} \\ = 2^{5a-4a+5b-4b+5c-4c} 5^{-2b+5b-2b-4c+10c-4c} = 2^{a+b+c} 5^{2c} 5^b$$

implies that $b \equiv 1 \pmod{2}$ if and only if $\chi(m) = (\frac{5}{m})$. Now, consider an eta quotient of the form

$$F = \eta^{a_1}(q) \eta^{a_2}(2q) \eta^{a_3}(4q) \eta^{a_4}(5q) \eta^{a_5}(10q) \eta^{a_6}(20q) \eta^{a_7}(25q) \eta^{a_8}(50q) \eta^{a_9}(100q).$$

Since

$$1^{a_1} 2^{a_2} 4^{a_3} 5^{a_4} 10^{a_5} 20^{a_6} 25^{a_7} 50^{a_8} 100^{a_9} = 2^{a_2+2a_3+a_5+2a_6+a_8+2a_9} 5^{a_4+a_5+a_6+2a_7+2a_8+2a_9},$$

F is in $M_4(\Gamma_0(100), \chi)$ if and only if $a_2 + a_5 + a_8 \equiv 0 \pmod{2}$, $a_4 + a_5 + a_6 \equiv 1 \pmod{2}$.

Now let ψ be the Dirichlet character mod 5 sending 2 to i , and consider the following Eisenstein

series:

$$E_4^{\psi^2,1}(z) = \sum_{n=1}^{+\infty} \left(\sum_{d|n} \psi^2\left(\frac{n}{d}\right) d^3 \right) q^n,$$

$$E_4^{1,\psi^2}(z) = 1 - \frac{8}{B_{4,\psi^2}} \sum_{n=1}^{+\infty} \left(\sum_{d|n} \psi^2(d) d^3 \right) q^n = 1 + \sum_{n=1}^{+\infty} \left(\sum_{d|n} \psi^2(d) d^3 \right) q^n.$$

$\{E_4^{\psi^2,1}(z), E_4^{1,\psi^2}(z)\}$ span the Eisenstein subspace of $M_4(\Gamma_0(5), (\frac{5}{d}))$, where $(\frac{5}{d})$ is the Kronecker character modulo 5.

Let \mathbb{Z}_{25}^* be the Dirichlet Group of invertible integers modulo 25. \mathbb{Z}_{25}^* is obviously generated by 2. Let γ be the Dirichlet character such that $\gamma(2) = -1$. Then it can be verified easily that γ is the induced Dirichlet character from the Kronecker character $(\frac{5}{n})$ modulo 5 and there are two Eisenstein series as newforms in $M_4(\Gamma_0(25), \gamma)$:

$$\begin{aligned} E251 &:= q + 9iq^2 - 28iq^3 - 73q^4 + 252q^6 + 344iq^7 - 585iq^8 - 757q^9 + 1332q^{11} + O(q^{12}) \\ E252 &:= q - 9iq^2 + 28iq^3 - 73q^4 + 252q^6 - 344iq^7 + 585iq^8 - 757q^9 + 1332q^{11} + O(q^{12}). \end{aligned}$$

Let \mathbb{Z}_{100}^* be the Dirichlet Group of invertible integers modulo 100. \mathbb{Z}_{100}^* is obviously generated by 51 and 77. Let α and β be the Dirichlet characters such that $\alpha(51) = -1$, $\alpha(77) = 1$ and $\beta(51) = 1$, $\beta(77) = -1$ respectively. Then it can be verified easily that β^{10} is the induced Dirichlet character from the Kronecker character $(\frac{5}{n})$ modulo 5. The dimension of the whole space $M_4(\Gamma_0(100))$ is 970. But here, we will only describe the subspace $M_4(\Gamma_0(100), \beta^{10})$. The dimension of $M_4(\Gamma_0(100), \beta^{10})$ is 54 and its Eisenstein subspace is 18 dimensional.

The set

$$\begin{aligned} &\{E_4^{1,\psi^2}(z), E_4^{1,\psi^2}(2z), E_4^{1,\psi^2}(4z), E_4^{1,\psi^2}(5z), E_4^{1,\psi^2}(10z), E_4^{1,\psi^2}(20z) \\ &E_4^{\psi^2,1}, E_4^{\psi^2,1}(2z), E_4^{\psi^2,1}(4z), E_4^{\psi^2,1}(5z), E_4^{\psi^2,1}(10z), E_4^{\psi^2,1}(20z), \\ &E251, E251(2z), E251(4z), E252, E252(2z), E252(4z), \end{aligned}$$

generates Eisenstein subspace of $M_4(\Gamma_0(100), \beta^{10})$.

Moreover, let

$$\begin{aligned}
 B_1(q) &:= \frac{\eta(2z)^3 \eta(4z)^5 \eta(5z)^4 \eta(50z)}{\eta(z)^3 \eta(20z) \eta(25z)}, B_2(q) := \frac{\eta(2z)^4 \eta(4z) \eta(5z)^4 \eta(50z) \eta(100z)}{\eta(z)^3 \eta(10z)^3 \eta(20z)^2 \eta(25z)}, \\
 B_3(q) &:= \frac{\eta(2z)^9 \eta(4z) \eta(5z)^3 \eta(50z)^4}{\eta(z)^5 \eta(10z) \eta(25z)^2 \eta(100z)}, B_4(q) := \frac{\eta(2z) 10 \eta(5z)^8 \eta(20z)^3 \eta(50z)^7}{\eta(z)^5 \eta(4z)^4 \eta(10z)^5 \eta(25z)^3 \eta(100z)^3}, \\
 B_5(q) &:= \frac{\eta(2z)^{10} \eta(5z)^6 \eta(20z)^4 \eta(50z)^2}{\eta(z)^5 \eta(4z)^3 \eta(10z)^4 \eta(25z) \eta(100z)}, B_6(q) = \frac{\eta(2z)^{10} \eta(5z)^6 \eta(20z) \eta(50z)^5}{\eta(z)^5 \eta(4z)^3 \eta(10z)^3 \eta(25z) \eta(100z)^2}, \\
 B_7(q) &:= \frac{\eta(2z)^{10} \eta(5z)^4 \eta(20z)^2}{\eta(z)^5 \eta(4z)^2 \eta(10z)^2 \eta(25z)}, B_8(q) = \frac{\eta(2z)^{10} \eta(5z)^6 \eta(100z)^2}{\eta(z)^5 \eta(4z)^2 \eta(10z)^2 \eta(25z)}, \\
 B_9(q) &:= \frac{\eta(2z)^{10} \eta(5z) \eta(10z)^3 \eta(100z)}{\eta(z)^5 \eta(4z) \eta(50z)}, B_{10}(q) := \frac{\eta(2z)^{10} \eta(5z)^2 \eta(25z)^3 \eta(100z)}{\eta(z)^5 \eta(4z) \eta(50z)^2}, \\
 B_{11}(q) &:= \frac{\eta(2z)^7 \eta(5z)^9 \eta(20z)^4 \eta(50z)^2}{\eta(z)^4 \eta(4z)^3 \eta(10z)^5 \eta(25z) \eta(100z)}, B_{12}(q) := \frac{\eta(2z)^6 \eta(5z)^9 \eta(50z) \eta(100z)}{\eta(z)^4 \eta(4z) \eta(10z)^3 \eta(25z)}, \\
 B_{13}(q) &:= \frac{\eta(2z)^6 \eta(5z)^9 \eta(50z)}{\eta(z)^4 \eta(10z)^3 \eta(25z)}, B_{14}(q) := \frac{\eta(2z)^6 \eta(5z)^6 \eta(50z)^4}{\eta(z)^4 \eta(4z) \eta(10z)^2 \eta(25z)^2 \eta(100z)}, \\
 B_{15}(q) &:= \frac{\eta(2z)^7 \eta(5z)^6 \eta(10z) \eta(50z)^4}{\eta(z)^4 \eta(4z)^2 \eta(20z) \eta(25z)^2 \eta(100z)}, B_{16}(q) := \frac{\eta(2z)^7 \eta(5z)^7 \eta(20z)^2 \eta(25z)}{\eta(z)^4 \eta(4z)^2 \eta(10z)^3}, \\
 B_{17}(q) &:= \frac{\eta(2z)^7 \eta(5z)^6 \eta(50z)^3 \eta(100z)}{\eta(z)^4 \eta(4z) \eta(10z)^2 \eta(25z)^2}, B_{18}(q) := \frac{\eta(2z)^7 \eta(5z)^4 \eta(100z)}{\eta(z)^4 \eta(4z) \eta(10z)^2 \eta(50z)}, \\
 B_{19}(q) &:= \frac{\eta(2z)^7 \eta(10z)^5 \eta(25z)}{\eta(z)^4 \eta(4z)^2 \eta(5z) \eta(20z)^2}, B_{20}(q) := \frac{\eta(2z)^7 \eta(5z)^6 \eta(50z)^3}{\eta(z)^4 \eta(10z)^2 \eta(25z)^2}, \\
 B_{21}(q) &:= \frac{\eta(2z)^7 \eta(5z) \eta(10z)^3 \eta(50z)^2}{\eta(z)^4 \eta(4z) \eta(25z) \eta(100z)}, B_{22}(q) := \frac{\eta(2z)^7 \eta(5z)^2 \eta(25z)^2 \eta(50z)}{\eta(z)^4 \eta(4z) \eta(100z)}, \\
 B_{23}(q) &:= \frac{\eta(2z)^8 \eta(5z)^6 \eta(50z)^2 \eta(100z)^3}{\eta(z)^4 \eta(4z)^3 \eta(10z)^2 \eta(25z)^2}, B_{24}(q) := \frac{\eta(2z)^8 \eta(5z) \eta(10z)^6 \eta(50z)^2}{\eta(z)^4 \eta(4z)^2 \eta(20z) \eta(25z) \eta(100z)}, \\
 B_{25}(q) &:= \frac{\eta(4z)^2 \eta(5z)^9 \eta(10z) \eta(50z)^3}{\eta(z)^2 \eta(20z)^2 \eta(25z)^3}, B_{26}(q) := \frac{\eta(4z)^4 \eta(5z)^7 \eta(20z) \eta(50z)^2}{\eta(z)^2 \eta(10z)^2 \eta(25z) \eta(100z)}, \\
 B_{27}(q) &:= \frac{\eta(2z)^2 \eta(4z)^2 \eta(5z)^9 \eta(10z) \eta(50z)}{\eta(z)^3 \eta(20z)^2 \eta(25z)^2}, B_{28}(q) := \frac{\eta(2z)^3 \eta(5z)^9 \eta(10z) \eta(100z)^2}{\eta(z)^3 \eta(20z)^2 \eta(25z)^2}, \\
 B_{29}(q) &:= \frac{\eta(2z)^3 \eta(4z) \eta(5z)^9 \eta(50z)^4}{\eta(z)^3 \eta(10z)^3 \eta(25z)^2 \eta(100z)}, B_{30}(q) := \frac{\eta(4z)^5 \eta(5z)^5 \eta(25z)}{\eta(z)^2 \eta(20z)}, \\
 B_{31}(q) &:= \frac{\eta(2z)^3 \eta(4z)^5 \eta(5z)^2 \eta(10z) \eta(25z)}{\eta(z)^3 \eta(20z)}, B_{32}(q) := \frac{\eta(2z)^5 \eta(10z)^7 \eta(20z)^2 \eta(100z)}{\eta(z) \eta(4z)^3 \eta(5z)^3}, \\
 B_{33}(q) &:= \frac{\eta(2z)^5 \eta(10z) \eta(20z)^6 \eta(50z)^6}{\eta(z) \eta(4z)^3 \eta(5z) \eta(25z)^2 \eta(100z)^3}, B_{34}(q) := \frac{\eta(2z)^6 \eta(10z)^5 \eta(25z)^2 \eta(50z)}{\eta(4z)^2 \eta(5z)^2 \eta(20z) \eta(100z)}, \\
 B_{35}(q) &:= \frac{\eta(2z)^8 \eta(5z)^2 \eta(20z) \eta(50z)^5}{\eta(4z)^4 \eta(10z) \eta(25z)^2 \eta(100z)}, \\
 B_{36}(q) &:= \frac{\eta(5z)^3 \eta(10z)^{10} \eta(20z)^9 \eta(25z)^3 \eta(50z)^8 \eta(100z)^5}{\eta(z)^{10} \eta(2z)^{10} \eta(4z)^{10}}.
 \end{aligned}$$

be some eta quotients of level 100 and weight 4 with nebentypus χ .

Theorem 3 The set

$$\begin{aligned} & \{E_4^{1,\psi^2}(z), E_4^{1,\psi^2}(2z), E_4^{1,\psi^2}(4z), E_4^{1,\psi^2}(5z), E_4^{1,\psi^2}(10z), E_4^{1,\psi^2}(20z) \\ & E_4^{\psi^2,1}, E_4^{\psi^2,1}(2z), E_4^{\psi^2,1}(4z), E_4^{\psi^2,1}(5z), E_4^{\psi^2,1}(10z), E_4^{\psi^2,1}(20z), \\ & E251, E251(2z), E251(4z), E252, E252(2z), E252(4z), \end{aligned}$$

is a basis of Eisenstein subspace of $M_4(\Gamma_0(100), \chi)$ and $\{B_1, B_2, \dots, B_{36}\}$ is a basis of cuspidal subspace of $M_4(\Gamma_0(100), \chi)$.

Proof $M_4(\Gamma_0(100), \chi)$ is 54 dimensional and $S_4(\Gamma_0(100), \chi)$ is 36 dimensional, see [16] (Chapter 3, pg.87 and Chapter 5, pg.197). So the first statement is clear. Second statement follows from lemma 1 and holomorphicity criterion in [[15] Corollary 2.3,p.37].

Corollary 4 Θ_Q can be expressed as linear combinations of the basis elements

$$\begin{aligned} & \{E_4^{1,\psi^2}(z), E_4^{1,\psi^2}(2z), E_4^{1,\psi^2}(4z), E_4^{1,\psi^2}(5z), E_4^{1,\psi^2}(10z), E_4^{1,\psi^2}(20z) \\ & E_4^{\psi^2,1}, E_4^{\psi^2,1}(2z), E_4^{\psi^2,1}(4z), E_4^{\psi^2,1}(5z), E_4^{\psi^2,1}(10z), E_4^{\psi^2,1}(20z), \\ & E251, E251(2z), E251(4z), E252, E252(2z), E252(4z), B_1, B_2, \dots, B_{36}\} \end{aligned}$$

for Q in the Table 1. The formulas are given in Table 2.

Theorem 5 Each B_i can be represented as linear combinations of newforms and their rescalings

$$\begin{aligned} & \Delta_{\chi,10,4,1}, \Delta_{\chi,10,4,1}(2z), \Delta_{\chi,10,4,1}(5z), \Delta_{\chi,10,4,1}(10z), \\ & \Delta_{\chi,10,4,2} \text{ (conjugate of } \Delta_{\chi,10,4,1} \text{ by } x^2 + 4\text{)}, \Delta_{\chi,10,4,2}(2z), \Delta_{\chi,10,4,2}(5z), \Delta_{\chi,10,4,2}(10z), \\ & \Delta_{\chi,20,4,1}(z), \Delta_{\chi,20,4,1}(5z), \\ & \Delta_{\chi,20,4,2}(z) \text{ (conjugate of } \Delta_{\chi,20,4,1} \text{ by } x^2 + 76\text{)}, \Delta_{\chi,20,4,2}(5z), \\ & \Delta_{\chi,25,4,1}(z), \Delta_{\chi,25,4,1}(2z), \Delta_{\chi,25,4,1}(4z), \\ & \Delta_{\chi,25,4,2}(z) \text{ (conjugate of } \Delta_{\chi,25,4,1} \text{ by } x^2 + 16\text{)}, \Delta_{\chi,25,4,2}(2z), \Delta_{\chi,25,4,2}(4z), \\ & \Delta_{\chi,25,4,3}(z), \Delta_{\chi,25,4,3}(2z), \Delta_{\chi,25,4,3}(4z), \\ & \Delta_{\chi,25,4,4}(z) \text{ (conjugate of } \Delta_{\chi,25,4,3} \text{ by } x^2 + 1\text{)}, \Delta_{\chi,25,4,4}(2z), \Delta_{\chi,25,4,4}(4z), \\ & \Delta_{\chi,50,4,1}(z), \Delta_{\chi,50,4,1}(2z), \\ & \Delta_{\chi,50,4,2}(z) \text{ (conjugate of } \Delta_{\chi,50,4,1} \text{ by } x^2 + 64\text{)}, \Delta_{\chi,50,4,2}(2z), \\ & \Delta_{\chi,50,4,3}(z), \Delta_{\chi,50,4,3}(2z), \\ & \Delta_{\chi,50,4,4}(z) \text{ (conjugate of } \Delta_{\chi,50,4,3} \text{ by } x^2 + 49\text{)}, \Delta_{\chi,50,4,4}(2z), \\ & \Delta_{\chi,100,4,1}(z), \Delta_{\chi,100,4,2}(z) \text{ (conjugate of } \Delta_{\chi,100,4,1} \text{ by } x^2 + 1\text{)}, \\ & \Delta_{\chi,100,4,3}(z), \Delta_{\chi,100,4,4}(z), \text{ (conjugate of } \Delta_{\chi,100,4,3} \text{ by } x^2 + 16\text{)} \end{aligned}$$

as in the table 3.

2 Conclusion

As a consequence of Table 2, we immediately get the following corollary:

$$\text{Let } \kappa(n) := \sum_{0 < d | n} \psi\left(\frac{n}{d}\right) d^3, \lambda(n) := -\frac{8}{B_{4,\psi}} \sum_{0 < d | n} \psi(d) d^3.$$

Corollary 6 The following formulas for the representation numbers are valid.

$$\begin{aligned}
 & + \frac{61760}{13}b_{12}(n) + \frac{24392}{1105}b_{13}(n) + \frac{30856}{65}b_{14}(n) - \frac{5600}{17}b_{15}(n) - \frac{25360}{221}b_{16}(n) \\
 & + \frac{92488}{221}b_{17}(n) - \frac{110936}{221}b_{18}(n) + \frac{119440}{221}b_{19}(n) + \frac{10144}{221}b_{20}(n) \\
 & + \frac{65152}{221}b_{21}(n) - \frac{69960}{221}b_{22}(n) - \frac{19144}{17}b_{23}(n) - \frac{19144}{17}b_{24}(n) \\
 & + \frac{834520}{221}b_{25}(n) + \frac{221}{17}b_{26}(n) + \frac{2240}{17}b_{27}(n) - \frac{7840}{17}b_{31}(n) + \frac{18320}{221}b_{33}(n).
 \end{aligned}$$

It is nice to obtain all representation numbers by means of Eisenstein series and eta quotients. Of course, in general it is not possible, [17]. But one can obtain similar formulas by applying method for all possible other cases.

All calculations have been done by MAGMA.

Competing Interests

Author has declared that no competing interests exist.

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