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# Effects of Viscous Dissipation Parameter on Magneto-hydrodynamic Free Convective Flow through a Porous Media of Parallel Plates with a Non-uniform Heat Source for an Unsteady Case

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***Authors' contributions***

*This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.*

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## Abstract

The magnetic hydrodynamic free convective flow past an infinite stretching porous sheet at constant density for electrically conducting fluid with viscous dissipation was numerically studied. The study revolved around an unsteady two-dimensional free convective laminar flow through a porous medium with the interaction of magnetic area standard to the stream. The graphs represented the effects of material parameters on the temperature and velocity profiles across the fluid boundary layer. The solutions of partial differential equations obtained numerically using an implicit finite difference method for various values of (nu), numbers (0.5 to 0.7) at a constant thermal conductivity (kappa=0.1). The velocity and temperature of MHD flow increased with an increase in viscous dissipation and vice versa.

*Keywords: Magnetohydrodynamic flow; finite-difference approximation; hydromagnetic flow.*

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## Acronyms

| | |
|---|---|
| $\vec{q}$ | *: Velocity of the Fluid* |
| $M$ | *: Coefficient of Viscosity* |
| $T$ | *: The Shear Stress* |
| $\frac{\partial U}{\partial T}$ | *: Temporal Acceleration* |
| $\mu \nabla^2 u$ | *: Viscous Term* |
| $u \nabla u$ | *: Convective Acceleration* |
| $\nabla P$ | *: Pressure Gradient* |
| $F$ | *: Body Force* |
| $K\ \nabla^2 T$ | *: Heat Due to Conduction* |
| $\mu^{\phi}$ | *: Viscous Term* |
| $C_P$ | *: Specific Heat Coefficient at Constant pressure* |
| $\frac{DT}{Dt}$ | *: Material Derivative* |
| $nu$ | *: Viscous Dispassion Parameters* |
| *MHD* | *: Magnetohydrodynamic* |
| *IFDM* | *: Implicit Finite Difference Method* |
| $\rho g$ | *: Gravitational Force* |
| $q$ | *: Velocity vector* $ms - 1$ |
| $J$ | *: Electric Current of Density* |
| $\rho_{\infty}$ | *: Free Stream Fluid Density (kgm*$^{-3}$*)* |
| $\nabla$ | *: Gradient Operators*$( i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} )$ |
| $\nabla^2$ | *: Laplacian Operator* $( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} )$ |
| $i\ , j, k$ | *: Unit Vectors in the Components* $x *, y *, z *$ |
| $K$ | *: Thermal Conductivity* |
| $g$ | *: Acceleration Due to Gravity ms*$^{-2}$ |

## 1 Introduction

Hydrodynamics is the study of fluid movement and the effects of the forces involved in the fluid flow [1]. Magnetohydrodynamics comprises the flow of electrically conducted fluid interacting with the magnetic field [2]. Magnetohydrodynamics is a combination of words; magneto, which means a magnetic area, Hydro implies water, dynamic meaning movement plus the forces that lead the motion [3]. Synonyms of MHD less frequently utilized are the terms of magnetofluid dynamics and hydromagnetics [4]. The field of MHD was initiated by the Swedish Physicist Hannes Alfvén (1908-1995), who received the Physics Nobel Prize in the year1970 for fundamental work and discoveries in magnetohydrodynamics with fruitful applications in different parts of plasma physics [5]. MHD covers those phenomena, where, in an electrically conducting fluid, the velocity field $V$and the magnetic field $B$ coupled. The magnetic field induces an electric current of density *j* in the moving conductive fluid (electromagnetism [6]. The induced flow creates forces on the liquid and changes the magnetic field. Each unit volume of the fluid having magnetic field $B$ experiences an MHD force $J \times B$ known as Lorentz force [7]. Any matter, which goes through deformation whenever forces are applied to it, however small, can be termed as fluid. Salty water, ionized gasses, liquid metals, electrolyte and plasma, are examples of electrically conducted fluid [8]. MHD takes place in earth interior compositions that produce a magnetic field as they move [9]. When electrically conducting fluid goes through a magnetic area, there is a generation of current to a conductor in the way conjointly at 90 degrees to both directions of fluid flow and field [10]. Fluid dynamics is the study of fluid, and the forces that cause motion, in fluid kinematics, the fluid parameters such as patterns of circulation, velocity and acceleration are studied [11].

## 1.1 Background information

The advancements of the MHD stream have attracted certain specialists previously in the past. Hence, this area contains the exploratory and hypothetical writing related to magnetohydrodynamic, the impact of speed and temperature of MHD flow-through channel with parallel plates at a constant density and unsteady stream. In fluid streams, viscous dissipation implies dissipation of energy or scattering of Vorticity. In a viscous fluid stream, the fluid viscosity takes power from the movement of the fluid (kinetic energy) and changes it into the internal energy of the liquid. It prompts warming up the fluid (viscous dissipation). At the point when an electrically conducting fluid streams, expansion in temperature prompts to a rise in its skin friction or viscosity. Viscosity alludes to the property of a liquid that decides its resistance to shearing stresses between the layers of fluid. It is a proportion of the internal friction of fluid making obstruction the liquid stream

Mathematically [12],

$$\tau = \mu \vec{q} \tag{1}$$

When an electrically conducting fluid flows through parallel plates, an increase in temperature leads to a rise in its skin-friction or viscosity. This viscosity increase could be along the $x - axis,$ the $y - axis$, or along the $z - axis$. The expression below illustrates the viscous dissipation in three-dimension,

$$\emptyset = \left\{ \left[\frac{\partial u}{\partial x}\right]^2 + \left[\frac{\partial v}{\partial y}\right]^2 + \left[\frac{\partial w}{\partial z}\right]^2 \right\} + \left\{ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial z} + \frac{\partial w}{\partial x} \right\}^2 \tag{2}$$

Equation 2 [13] can be reduced to two dimensions to give;

$$\emptyset = \left\{ \left[\frac{\partial u}{\partial x}\right]^2 + \left[\frac{\partial v}{\partial y}\right]^2 \right\} + \left\{ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right\}^2 \tag{3}$$

Equation 3 is simplified, the final form of the viscous dissipation term is;

$$\emptyset = \left\{ \left[\frac{\partial u}{\partial x}\right]^2 + 2\frac{\partial v \partial u}{\partial x \partial y} + \left[\frac{\partial v}{\partial y}\right]^2 + \left[\frac{\partial u}{\partial y}\right]^2 + \left[\frac{\partial v}{\partial x}\right]^2 \right\} \tag{4}$$

Rashidi, Rostami, Freidoonimehr, & Abbasbandy, (2014) researched the steady flow of a viscous incompressible electrically leading liquid over a level sheet they discovered that the temperature profiles rely upon the thickness of the fluid [14]. Contreras and the associates examined numerically magneto-hydrodynamic MHD transient natural convection-radiation limit layer flow with variable surface temperature. Demonstrating that ascending in radiation parameter increments, and the Nusselt number stands decreased upgrading speed, heat, and skin friction [15].

Veera Krishna and Suneetha examined Hall consequences for the unsteady progression of incompressible viscous fluid between two rigid non-conducting pivoting plates through a porous medium that is affected by a uniform transverse magnetic field [16]. The outcomes demonstrated that temperature circulation in the fluid movement and the Nusselt number got impacted by the stream structures with the side walls. Ibrahim, Elaiw, and Bakr (2008) directed the effect of the chemical reaction and radiation absorption on the unsteady MHD free convection stream past a semi-boundless vertical porous moving plate with a heat source and suction [17].

Kinyanjui et al. (1998) investigated Hall's current impact on magnetohydrodynamic free-convection stream past a semi-endless vertical plate with mass exchange [18]. They talked about the effects of the magnetic parameter, hall parameter, and the relative lightness power impact among species and warm dissemination on the speed, temperature, and concentration. Pantokratoras (2009) studied the idea of vertical standard convection streams coming about because of the consolidated buoyancy impacts of thermal and mass diffusion [19]. The skin-rubbing factor for the tangential flow and the Nusselt number declines; however, the skin friction factor for the lateral stream increments as the magnetic field increments.

## 2 Methodology

The equations governing MHD flows through a porous parallel infinite plate were Maxwell equations, momentum equation, continuity equation, and energy equation. We also carried out non-dimensionalization of resulting simplified equations and their initial boundary conditions. The equations governing our free convective fluid flow in our study were non-linear; therefore, the finite difference approximation method used to solve the equations as it is fast and stable. The difference method used should satisfy the basic requirements of consistency, stability, and convergence. This method was convergent since taking more grid points, or step size decreased, and the numerical solution converges to the exact solution, it was also stable as the effect of any single fixed round off error bounds. Finally, the method was consistent as the truncation error tended to zero as the step size decreases.

In fact, in some cases, the exact solutions differed considerably from different solutions. If the effects of the round off error remained bounded as the mesh point tended to infinity with step size, then the difference method was said to be stable. The MHD flow was horizontal (along the x-axis), while the magnetic area applied was normal to the fluid flow (along the y-axis). The diagram below illustrates the movement of the MHD;

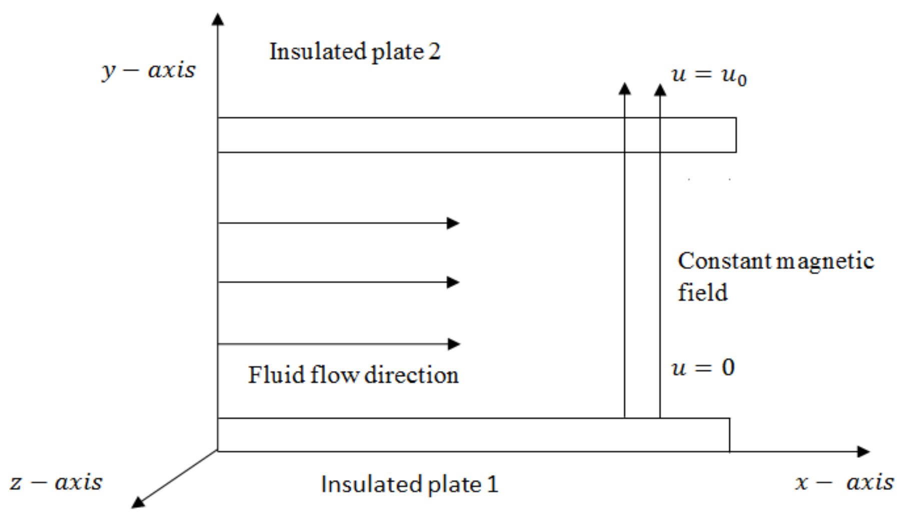


**Fig. 1. Schematic diagram of the flow at both plates stationary (author)**

In an infinite parallel plate, the boundary plates were assumed to be infinite in extent, both in the direction from left to right. This assumption was necessary so that the "end effects neglected ". The edges of the plates were quite distant from the portion of the fluid under consideration. The diagram below shows a fully developed fluid flow situation.

The general equations that are governing the flow of electrically conducted fluid and incompressible fluid in the presence of a uniform magnetic field were the equation of momentum, continuity equation, and the equation of conservation of energy, Maxwell's equations, and Ohm's law. We also considered the general equations governing magnetic hydrodynamic free convective flows past infinite plates with viscous dissipation. The simplified hypothesis was also spelled out, which were very crucial in finding the solutions to the equations arrived at in this study.

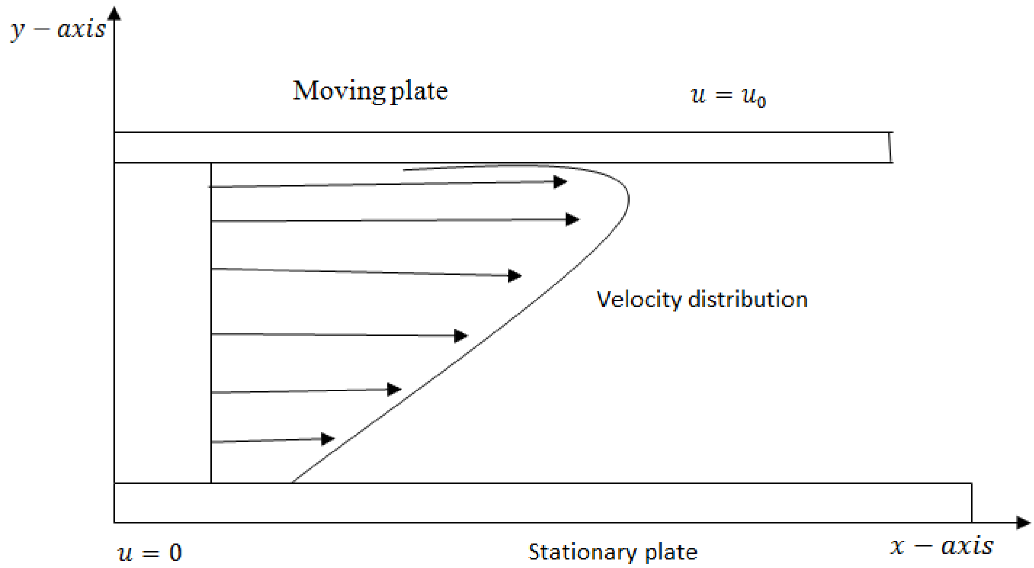


**Fig. 2. Illustration of the flow where the upper plate is moving and lower plate stationary.**

# 3 Specific Equations Governing the MHD Flow

## 3.1 Equation of continuity

If $\vec{q} = [u, v]$ is the velocity of the fluid whose density is $\rho$, then the motion is possible if,

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}.\rho\vec{q} = 0 \quad (5)$$

Equation (5) implies that mass was conserved and that for any liquid, the flow was assumed to be continuous, that is, no empty spaces occur between particles which were in contact.
A liquid is a fluid whose volume does not change with changes in pressure, and if the change occurs, it's negligible; thus, a liquid is said to be incompressible when the density, $\rho$, of a liquid is assumed to be constant, equation (5) becomes;

$$\overrightarrow{\nabla.(\rho\vec{q})} = 0 \quad (6)$$

By product rule of differentiation, equation (6) becomes;

$$[\vec{\nabla}\rho].\vec{q} + \rho[\vec{\nabla}.\vec{q}] = 0 \quad (7)$$

$$\rho[\vec{\nabla}.\vec{q}] = 0 \quad (8)$$

Since $\rho \neq 0$, then

$$\vec{\nabla}.\vec{q} = 0 \quad (9)$$

In three dimensions the continuity equation was expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (10)$$

considering the flow was passed through an infinite vertical porous plate, and the flow variables were only depending on the $x$ and $y - axes$; hence equation (10) become,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (11)$$

Since the plates were parallel to the $x - axis$ and infinite porous plates in extent the velocity depended on y only which implied that, there was no flow in the $y$ −direction,

$$v = 0.$$

$$\frac{\partial v}{\partial y} = 0 \quad (12)$$

Thus the equation of continuity (6) above was satisfied; hence, the motion of the fluid occurred.

## 3.2 Momentum equation

Considering this equation was based on the law of conservation of the momentum, the net sum of the forces acting on MHD fluid should be the same as the net flow rate of MHD fluid. Obtaining momentum equation was by taking each term in the Navier-Stokes equation with the vector of velocity $u$. It resulted in the form of expression of a change in the time rate of energy in motion of MHD fluid per unit volume. The equation of momentum in tensor form as shown below;

$$\rho\left(\frac{\partial u}{\partial t} + u\nabla u\right) = \rho F - \nabla \mathrm{P} + \mu\nabla^2 u + J \times B + \rho g \quad (13)$$

The external body forces were due to gravitational force $\rho$g, and the electromagnetic energy was given by;

$$\vec{J} \times \vec{B} \quad (14)$$

For a two-dimensional fluid flow in component form, the momentum equation (13) written in component form in $i$ and $j$ as shown below;

$$\rho\frac{\partial u}{\partial t} + \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \vec{J} \times \vec{B} + \rho \mathrm{g}_x \quad (15)$$

$$\rho\frac{\partial v}{\partial t} + \rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \vec{J} \times \vec{B} + \rho \mathrm{g}_y \quad (16)$$

With

$$F = \rho \mathrm{g} + J \times \beta \quad (17)$$

We considered both the gravitational force and electromagnetic force to get the volumetric density of the external force.

For parallel flow with the plates infinite in extent in the x-direction, the velocity profiles depended only on the y-coordinates; thus v=0 and the pressure in the flow depended only on x-coordinates. Thus, equation (15) reduces to

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial P}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} + \vec{J} \times \vec{B} + \rho g_x \tag{18}$$

Reducing equation (16) to zero.

Since we assumed that the pressure gradient was constant and the force of gravity along the x-axis was zero, then equation (18) reduced to

$$\rho \frac{\partial u}{\partial t} = -P + \mu \frac{\partial^2 u}{\partial y^2} + \vec{J} \times \vec{B} \tag{19}$$

From the assumption that there was no external applied electric field, i.e., J =0 hence equation (19) reduced to

$$\rho \frac{\partial u}{\partial t} = -P + \mu \frac{\partial^2 u}{\partial y^2} \tag{20}$$

## 3.3 Equation of energy

The term viscous dispassion is found in the equation. Viscosity and temperature affect the general energy equation for MHD flows through an infinite parallel porous plate. Below was the general energy equation;

$$\rho C_P \frac{DT}{Dt} = K \nabla^2 T + \mu x + {j^2}/{\delta} \tag{21}$$

Or

$$\rho C_P \frac{DT}{Dt} = K \nabla^2 T + \mu^{\phi} + {j^2}/{\delta} \tag{22}$$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + q \nabla T$$

expressing equation (22) in two dimensions

$$\rho C_P \left[\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y}\right] = K \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] + \mu \left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right\} \tag{23}$$

Equation (23) reduced to the following on dividing by $\rho c_P$ throughout;

$$\left[\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x}\right] = \frac{k}{\rho C_p}\left[\frac{\partial^2 T}{\partial x^2}\right] + \frac{\mu}{\rho C_p}\left\{3\left[\left(\frac{\partial u}{\partial x}\right)^2\right] + \left(\frac{\partial u}{\partial y}\right)^2\right\} \tag{24}$$

In x co-ordinates

$$\left[\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y}\right] = \frac{k}{\rho C_p}\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] + \frac{\mu}{\rho C_p}\left\{2\left[\left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial x}\right)^2\right\} \tag{25}$$

In y co-ordinates.

Since the fluid flow was parallel to axis $y = 0$ and the plates were infinite in extent, $v = 0$ and no flow variable was a function of x. thus equation (24) becomes

$$\rho C_P\left(\frac{\partial T}{\partial t}\right)=k\left(\frac{\partial^2 T}{\partial y^2}\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2 \quad (26)$$

Two equations raised from the consideration of both the specific expression of motion and equation of energy were as follow;

$$\rho\left(\frac{\partial u}{\partial t}\right)=-P+\mu\left(\frac{\partial u}{\partial y}\right)^2 \quad (27)$$

and

$$\rho C_P\left(\frac{\partial T}{\partial t}\right)=k\left(\frac{\partial^2 T}{\partial y^2}\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2 \quad (28)$$

the initial and boundary conditions as

$u\ (0,y)=0\ T(0,y)=0$
$u\ (t,0)=u0\ T(t,0)=0$
$u\ (t,y\infty)=\ u0\ T(t,y\infty)\ =T0\ t>0$

# 4 Results and Discussion

considering equations 27 and 28.

$$\rho\left(\frac{\partial u}{\partial t}\right)=-P+\mu\left(\frac{\partial u}{\partial y}\right)^2 \quad (29)$$

And

$$\rho C_P\left(\frac{\partial T}{\partial t}\right)=k\left(\frac{\partial^2 T}{\partial y^2}\right)+\mu\left(\frac{\partial u}{\partial y}\right)^2 \quad (30)$$

We used finite difference method with a uniform spatial step $(n)$ and time step $(m)$. Taylor's series of a function of two variables $fx_0+t_O$ and $fx_0-t_O$ and for sufficiently small $h$, gives:

$$f(x_0+h,t_0\ )=\mathrm{f}(x_0,t_0)+\mathrm{hf}'(\mathrm{x})+\frac{\mathrm{h}^2}{2}\mathrm{f}''(\mathrm{x})+\frac{\mathrm{h}^3}{6}\mathrm{f}'''(\mathrm{x})+\cdots \quad (31)$$

$$f(x_0-h,t_0\ )=f(x_0,t_0)-hf'(x)+\frac{h^2}{2}f''(x)-\frac{h^3}{6}f'''(x)+\cdots \quad (32)$$

Replacing $f$ with $u$ the series become

$$(U_{X0+t0}+h,t_0)=\mathrm{u}(x_0,t_0)+\mathrm{hu}'(\mathrm{x})+\frac{\mathrm{h}^2}{2}\mathrm{u}''(\mathrm{x})+\frac{\mathrm{h}^3}{6}\mathrm{u}'''(\mathrm{x})+\cdots \quad (33)$$

and

$$\left(U_{x0-h,t0},\right)=(Ux_0,t_0)-\mathrm{hu}'(\mathrm{x})+\frac{\mathrm{h}^2}{2}\mathrm{u}''(\mathrm{x})-\frac{\mathrm{h}^3}{6}\mathrm{u}'''(\mathrm{x})+\cdots \quad (34)$$

Setting, $(U_{X0+h},t_0)=(U_{i+1},j)$ and

$$U_{X0-h,t0}=U_{i-1,j}$$

Equation 33 can be written as;

$$\left(U_{i+1,j}\right) = U_{i,j} + \mathrm{hu}'(\mathrm{x}) + \frac{\mathrm{h}^2}{2}\mathrm{u}''(\mathrm{x}) + \frac{\mathrm{h}^3}{6}\mathrm{u}'''(\mathrm{x}) + \cdots \quad (35)$$

And 34 as

$$\left(U_{i-1,j}\right) = U_{i,j} - \mathrm{hu}'(\mathrm{x}) + \frac{\mathrm{h}^2}{2}\mathrm{u}''(\mathrm{x}) - \frac{\mathrm{h}^3}{6}\mathrm{u}'''(\mathrm{x}) + \cdots \quad (36)$$

From equation 35

$$\mathrm{u}' = \frac{U_{i+1.j} - U_{i,j}}{h} \quad (37)$$

Where, $\mathrm{u}' = U_x = \frac{dU}{dx}$

from equation 36

$$\mathrm{u}' = \frac{U_{i..j} - U_{i-1.j}}{h} \quad (38)$$

by adding equation 33 equation 34

$U_{\ i+1,j} + U_{i-1,j}\ -2U_{i,j}$=$\mathrm{h}^2\mathrm{u}''$ by rearranging

$$U_{XX} = \frac{1}{\mathrm{h}^2}(U_{i-1,j} - 2U_{i,j} + U_{i+1,j}) \quad (39)$$

By subtracting equation 34 from equation 33

$$U_{X=}\frac{U_{i+1..j} - U_{i-1.j}}{2h} \quad (40)$$

Since MHD flow considered in two dimensions, the partial differential equations become,

$$U_{XX} = \frac{1}{\mathrm{h}^2}(U_{i-1,j} - 2U_{i,j} + U_{i+1,j}) \quad (41)$$

$$U_{X=}\frac{U_{i+1,j} - U_{i-1,j}}{2h} \quad (42)$$

$$T_{XX} = \frac{1}{\mathrm{h}^2}(T_{i-1,j} - 2T_{i,j} + T_{i+1,j}) \quad (43)$$

$$T_{X=}\frac{T_{i+1,j} - T_{i-1,j}}{2h} \quad (44)$$

And the forward difference for derivatives with respect to time

$$U_{t=}\frac{U_{i,j+1} - U_{i,j-1}}{2n} \quad (45)$$

$$T_{t=}\frac{T_{i,j+1}-T_{i,j-1}}{2n} \tag{46}$$

Discretizing equations 27 and 28 using forward differences for time and central differences for special variables y.

From 36 $\frac{\partial u}{\partial t}=\frac{u_{i,j+1}-u_{i,j}}{\Delta t}$ and $\frac{\partial^2 u}{\partial y^2}=\frac{1}{(\Delta y)^2}(U_{i-1,j}-2U_{i,j}+U_{i+1,j})$

Therefore

$$\frac{u_{i,j+1}-u_{i,j}}{\Delta t}=\frac{\acute{Y}}{(\Delta y)^2}(U_{i-1,j}-2U_{i,j}+U_{i+1,j}) \tag{47}$$

where $\acute{Y}=\frac{\mu}{\rho}$

Expressing equation 27 in terms of $u_{i,j+1}$ to give

$$u_{i,j+1}=\frac{\Delta t\,\acute{Y}}{(\Delta y)^2}(U_{i-1,j}-2U_{i,j}+U_{i+1,j})+U_{i,j} \tag{48}$$

Where $\frac{\Delta t\,\acute{Y}}{(\Delta y)^2}=r_1$

From equation 28

$$\frac{\partial T}{\partial t}=\frac{T_{i,j+1}-T_{ij}}{\Delta t} \tag{49}$$

$$\frac{\partial^2 T}{\partial t^2}=\frac{1}{(\Delta t^2)}\left(T_{i-1,j}-2T_{ij}+T_{i+1,j}\right) \tag{50}$$

$$\left(\frac{\partial u}{\partial y}\right)^2=\left(\frac{u_{i+1,j}-u_{i-1j}}{2\Delta y}\right)^2 \tag{51}$$

Therefore

$$\frac{T_{i,j+1}-T_{i,j}}{\Delta t}=\frac{k}{\rho(\Delta y)^2 C_p}(T_{i-1,j}-2T_{i,j}+T_{i+1,j})+\frac{\mu}{\rho C_p}\left(\frac{U_{i+1..j}-U_{i-1.j}}{2\Delta y}\right)^2 \tag{52}$$

$$T_{i,j+1}=\frac{\Delta t\times \text{kappa}}{\rho C_P(\Delta y)^2}(T_{i-1,j}-2T_{i,j}+T_{i+1,j})+\frac{\Delta t*\mu}{4\rho C_P(\Delta y)^2}(U_{i+1,j}-U_{i-1,j})^2-T_{i,j} \tag{53}$$

Where, $r_2=\frac{\Delta t\times \text{kappa}}{\rho C_P(\Delta y)^2}$, $r_3=\frac{\Delta t*\mu}{4\rho C_P(\Delta y)^2}$ and $\rho=rho$

To give the relationship between the partial derivatives in the differential equation and the function value at the adjacent nodal points, we, therefore, used a uniform mesh. In this case, the $t-y$ plane is divided into mesh points of uniform rectangular cells of width and height m and n, respectively. The $U$ values in equation (47) were found in every node point for a specific $i$.

Dividing the physical flow domain into a finite number for time and space domains that are discreetly approximated by finite difference method. From Fig. 3 plane, cell corners form grid points or the mesh point.

Points $(i, k)$ Were the reference points where $i$ and $k$ represent $t$ and $y$, where $i$ and $k$ represent $t$ and $y$, respectively. Setting the notation $(i + 2.5)$ for $(t + \Delta t)$ and $(k + 2)$ for $(y + \Delta y)$ the adjacent points i and k are defined units from the reference point and their co-ordinates given in terms of $\Delta t$ and $(i)$ along $y = 0$ and $\Delta y$ and $k$ along $x = 0$. The derivatives in the finite difference approximation substituted with the finite difference $U = u(t, y)$ and the intersection point of these grid points or mesh points . Then by the central difference. A finite-difference mesh is used to express the unknown function values at the $(i, k)^{th}$ interior mesh using the known boundary points. The finite difference analogues of the partial differential equations generated from the governing MHD equations obtained by substituting the derivatives in the governing equations by the difference approximation that are corresponding the boundary values and initial values set. If we set $n = 2.5$, and $m = 2$, where n represents a small change in y and m represents a slight change in $t$. We place $i$ and $k$ counter to range from 0 to 5 and 0 to 6 consecutively in positive directions.

**Table 1. Velocity versus temperature at constant distance**

| **Time ($t_{(k)}$)** | **Distance along x axis ($y_{(i)}$)** | **Initial velocity in x and y axis ($u_1(i, k)$)** | **Temperature in x and y axis ($T_1(i, k)$)** |
|---|---|---|---|
| 0.0000 | 0.0000 | 0.5000 | 0.0000 |
| 0.0000 | 2.5000 | 0.0000 | 0.0000 |
| 0.0000 | 5.0000 | 0.0000 | 0.0000 |
| 0.0000 | 7.5000 | 0.0000 | 0.0000 |
| 0.0000 | 10.0000 | 0.5000 | 0.5000 |
| 2.0000 | 0.0000 | 0.5000 | 0.0000 |
| 2.0000 | 2.5000 | 0.1250 | 0.2754 |
| 2.0000 | 5.0000 | 0.0000 | 0.0000 |
| 2.0000 | 7.5000 | 0.1250 | 0.2867 |
| 2.0000 | 10.0000 | 0.5000 | 0.5000 |
| 4.0000 | 0.0000 | 0.5000 | 0.0000 |
| 4.0000 | 2.5000 | 0.1875 | 0.5383 |
| 4.0000 | 5.0000 | 0.0625 | 0.0128 |
| 4.0000 | 7.5000 | 0.1875 | 0.5605 |
| 4.0000 | 10.0000 | 0.5000 | 0.5000 |
| 6.0000 | 0.0000 | 0.5000 | 0.0000 |
| 6.0000 | 2.5000 | 0.2344 | 0.7250 |
| 6.0000 | 5.0000 | 0.1250 | 0.0371 |
| 6.0000 | 7.5000 | 0.2344 | 0.7575 |
| 6.0000 | 10.0000 | 0.5000 | 0.5000 |
| 8.0000 | 0.0000 | 0.5000 | 0.0000 |
| 8.0000 | 2.5000 | 0.2734 | 0.8478 |
| 8.0000 | 5.0000 | 0.1797 | 0.0691 |
| 8.0000 | 7.5000 | 0.2734 | 0.8902 |
| 8.0000 | 10.0000 | 0.5000 | 0.5000 |
| 10.0000 | 0.0000 | 0.5000 | 0.0000 |
| 10.0000 | 2.5000 | 0.3066 | 0.9239 |
| 10.0000 | 5.0000 | 0.2266 | 0.1054 |
| 10.0000 | 7.5000 | 0.3066 | 0.9757 |
| 10.0000 | 10.0000 | 0.5000 | 0.5000 |

## 4.1 Viscous dissipation parameter on MHD free convective flow

The effects of viscous dissipation on MHD free convective flow vertical porous infinite plate has been carried out. The tables and graphs have been used to represent the result.

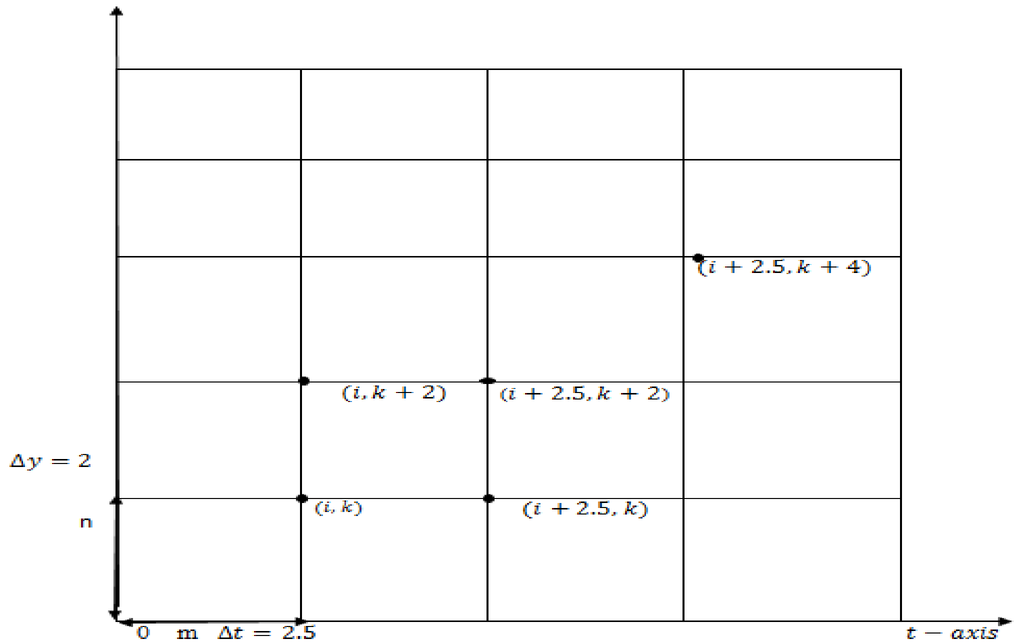


**Fig. 3. Illustration of mesh points**

From Table 1, the velocity at initial point remained constant until the temperature started varying. At a constant distance the velocity and temperature values increases with an increase time. From this table $U1$ and $T1$ were derived as the values of $k$, $i$, time and y were varied.

$k = 1\ 2\ 3\ 4\ 5\ 6$
$i = 1\ 2\ 3\ 4\ 5$

$U_1$ values have been derived from varied time values and distance values as shown below

| **Time Values =** | **0.0000** | **2.5000** | **5.0000** | **7.5000** | **10.0000** |
|---|---|---|---|---|---|
| **y-values =** | | | | | |
| 0 | 0.5000 | 0.0000 | 0.0000 | 0.0000 | 0.5000 |
| 2 | 0.5000 | 0 .1250 | 0.0000 | 0 .1250 | 0.5000 |
| 4 | 0.5000 | 0.1875 | 0.0625 | 0.1875 | 0.5000 |
| 6 | 0.5000 | 0.2344 | 0.1250 | 0.2344 | 0.5000 |
| 8 | 0.5000 | 0.2734 | 0.1797 | 0.2734 | 0.5000 |
| 10 | 0.5000 | 0.3066 | 0.2266 | 0.3066 | 0.5000 |

The velocity of the flow increases with time as the time increase; this occurred at a different length. On the other hand, the speed changes slightly with distance at a constant time.

| **Time Values =** | **0.0000** | **2.5000** | **5.0000** | **7.5000** | **10.0000** |
|---|---|---|---|---|---|
| **y-values =** | | | | | |
| 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.5000 |
| 2 | 0.0000 | 0.2754 | 0.0000 | 0.2867 | 0.5000 |
| 4 | 0.0000 | 0.5383 | 0.0128 | 0.5605 | 0.5000 |
| 6 | 0.0000 | 0.7250 | 0.0371 | 0.7575 | 0.5000 |
| 8 | 0.0000 | 0.8478 | 0.0691 | 0.8902 | 0.5000 |
| 10 | 0.0000 | 0.9239 | 0.1054 | 0.9757 | 0.5000 |

Temperature increase with an increase in time as the distance got varied.

**Table 2. Temperature versus time as viscous dispassion was varied**

| **Time $t_{(k)}$** | **Distance $y_{(i)}$** | **Velocity $u_3$ $(i,k)$)** | **Temperature $T_{\ 3}$ $(i,k)$** |
|---|---|---|---|
| 0.0000 | 0.5000 | 0.0000 | 0.0000 |
| 0.0000 | 2.5000 | 0.0000 | 0.0000 |
| 0.0000 | 5.0000 | 0.0000 | 0.0000 |
| 0.0000 | 7.5000 | 0.0000 | 0.0000 |
| 0.0000 | 10.0000 | 0.5000 | 0.5000 |
| 2.0000 | 0.0000 | 0.5000 | 0.0000 |
| 2.0000 | 2.5000 | 0.1500 | 0.6649 |
| 2.0000 | 0.5000 | 0.0000 | 0.0000 |
| 2.0000 | 7.5000 | 0.1500 | 0.6762 |
| 2.0000 | 10.0000 | 0.5000 | 0.5000 |
| 4.0000 | 0.0000 | 0.5000 | 0.0000 |
| 4.0000 | 2.5000 | 0.2100 | 1.2996 |
| 4.0000 | 5.0000 | 0.0900 | 0.0304 |
| 4.0000 | 7.5000 | 0.2100 | 1.3218 |
| 4.0000 | 10.0000 | 0.5000 | 0.5000 |
| 6.0000 | 0.0000 | 0.5000 | 0.0000 |
| 6.0000 | 2.5000 | 0.2610 | 1.6884 |
| 6.0000 | 5.0000 | 0.1620 | 0.0885 |
| 6.0000 | 7.5000 | 0.2610 | 1.7209 |
| 6.0000 | 10.0000 | 0.5000 | 0.5000 |
| 8.0000 | 0.0000 | 0.5000 | 0.0000 |
| 8.0000 | 2.5000 | 0.3030 | 1.9176 |
| 8.0000 | 5.0000 | 0.2214 | 0.1619 |
| 8.0000 | 7.5000 | 0.3030 | 1.9600 |
| 8.0000 | 10.0000 | 0.5000 | 0.5000 |
| 10.0000 | 0.0000 | 0.5000 | 0.0000 |
| 10.0000 | 2.5000 | 0.3376 | 2.0407 |
| 10.0000 | 5.0000 | 0.2704 | 0.2426 |
| 10.0000 | 7.5000 | 0.3376 | 2.0925 |
| 10.0000 | 10.0000 | 0.5000 | 0.5000 |

From the Table 2 we come up with values of velocity and temperature as the values of time and $(t)$ and distance are varied the values are as follow,

$k\ = 1\ 2\ 3\ 4\ 5\ 6$
$i\ = 1\ 2\ 3\ 4\ 5$

$U3$ obtained from the varying time and distances values have been illustrated below

| **Time Values =** | **$0.0000$** | **2.5000** | **5.0000** | **7.5000** | **10.0000** |
|---|---|---|---|---|---|
| **y-values =** | | | | | |
| 0 | 0.5000 | 0.0000 | 0.0000 | 0.0000 | 0.5000 |
| 2 | 0.5000 | 0.1500 | 0.0000 | 0.1500 | 0.5000 |
| 4 | 0.5000 | 0.2100 | 0.0900 | 0.2100 | 0.5000 |
| 6 | 0.5000 | 0.2610 | 0.1620 | 0.2610 | 0.5000 |
| 8 | 0.5000 | 0.3030 | 0.2214 | 0.3030 | 0.5000 |
| 10 | 0.5000 | 0.3376 | 0.2704 | 0.3376 | 0.5000 |

At zero distance ($y = 0$) velocity don't vary with time, as the distance ($y$) started varying the velocity $U_3$increase with an increase in time ($t$). the values of the velocity of the fluid at $y = 0$ and $y = 10$ remains constant while the time is varied. Thermal conductivity remaining constant ($kappa = 0.1$)

The values of temperature ($T3$) were shown below as the values of time were varied from $0 - 10$ at intervals of 2.5 and distance (y-values) from $0 - 10$ at an intervals of 2.

| **Time Values =** | **0.0000** | **2.5000** | **5.0000** | **7.5000** | **10.0000** |
|---|---|---|---|---|---|
| **y-values=** | | | | | |
| 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.5000 |
| 2 | 0.0000 | 0.6649 | 0.0304 | 0.6762 | 0.5000 |
| 4 | 0.0000 | 1.2996 | 0.0128 | 1.3218 | 0.5000 |
| 6 | 0.0000 | 1.6884 | 0.0885 | 1.7209 | 0.5000 |
| 8 | 0.0000 | 1.9176 | 0.1619 | 1.9600 | 0.5000 |
| 10 | 0.0000 | 2.0407 | 0.2426 | 2.0925 | 0.5000 |

At the boundary conditions the $y = 0$ and $y = 10$ the temperature of the fluid

$T(3)$ remains unvaried while the values of time ($t$) were varied. The temperature varies with distance at a specific time values. From these values, it indicates an increase in time ($t$) leads to an increase in temperature.

**Table 3. Velocity versus time varying $nu$ values**

| **Time $t_{(k)}$** | **Distance $y_{(i)}$** | **Velocity $U_5\,(i,k)$** | **Temperature $T_5(i,k)$** |
|---|---|---|---|
| 0.0000 | 0.0000 | 0.5000 | 0.0000 |
| 0.0000 | 2.5000 | 0.0000 | 0.0000 |
| 0.0000 | 0.5000 | 0.0000 | 0.0000 |
| 0.0000 | 7.5000 | 0.0000 | 0.0000 |
| 0.0000 | 10.0000 | 0.5000 | 0.5000 |
| 2.0000 | 0.0000 | 0.5000 | 0.0000 |
| 2.0000 | 2.5000 | 0.1750 | 0.7757 |
| 2.0000 | 0.5000 | 0.0000 | 0.0000 |
| 2.0000 | 7.5000 | 0.1750 | 0.7871 |
| 2.0000 | 10.0000 | 0.5000 | 0.5000 |
| 4.0000 | 0.0000 | 0.5000 | 0.0000 |
| 4.0000 | 2.5000 | 0.2275 | 1.5162 |
| 4.0000 | 5.0000 | 0.1225 | 0.0355 |
| 4.0000 | 7.5000 | 0.2275 | 1.5384 |
| 4.0000 | 10.0000 | 0.5000 | 0.5000 |
| 6.0000 | 0.0000 | 0.5000 | 0.0000 |
| 6.0000 | 2.5000 | 0.2861 | 1.8904 |
| 6.0000 | 5.0000 | 0.1960 | 0.1032 |
| 6.0000 | 7.5000 | 0.2861 | 1.9229 |
| 6.0000 | 10.0000 | 0.5000 | 0.5000 |
| 8.0000 | 0.0000 | 0.5000 | 0.0000 |
| 8.0000 | 2.5000 | 0.3294 | 2.0937 |
| 8.0000 | 5.0000 | 0.2591 | 0.1850 |
| 8.0000 | 7.5000 | 0.3294 | 2.1360 |
| 8.0000 | 10.0000 | 0.5000 | 0.5000 |
| 10.0000 | 0.0000 | 0.5000 | 0.0000 |
| 10.0000 | 2.5000 | 0.3645 | 2.1829 |
| 10.0000 | 5.0000 | 0.3083 | 0.2726 |
| 10.0000 | 7.5000 | 0.3645 | 2.2347 |
| 10.0000 | 10.0000 | 0.5000 | 0.5000 |

We get the values of velocity $U_5$ and $T_5$ from the Table 3

$k\ = 1\ 2\ 3\ 4\ 5\ 6$
$i\ =\ 1\ 2\ 3\ 4\ 5$

time Values = 0 2 4 6 8 10

$Y - values$ =0 2.5000 5.0000 7.5000 10.0000
$U_5 values\ =$

| Time Values = | 0.0000 | 2.5000 | 5.0000 | 7.5000 | 10.0000 |
|---|---|---|---|---|---|
| y-values = | | | | | |
| 0 | 0.5000 | 0.0000 | 0.0000 | 0.0000 | 0.5000 |
| 2 | 0.5000 | 0.1750 | 0.1225 | 0.1750 | 0.5000 |
| 4 | 0.5000 | 0.2275 | 0.0128 | 0.2275 | 0.5000 |
| 6 | 0.5000 | 0.2861 | 0.1960 | 0.2861 | 0.5000 |
| 8 | 0.5000 | 0.3294 | 0.2591 | 0.3294 | 0.5000 |
| 10 | 0.5000 | 0.3645 | 0.3083 | 0.3645 | 0.5000 |

From $(U5)$ values it is noted that, velocity remains constant at $y = 0$ when time is varied. Increase in time $(\ t\ )$ values leads to enhanced acceleration of the fluid flow as the distance changes hence increasing the velocity.

$T_5 values\ =$

| Time Values = | 0.0000 | 2.5000 | 5.0000 | 7.5000 | 10.0000 |
|---|---|---|---|---|---|
| y-values = | | | | | |
| 0 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.5000 |
| 2 | 0.0000 | 0.7757 | 0.0000 | 0.7871 | 0.5000 |
| 4 | 0.0000 | 1.5162 | 0.0355 | 1.5384 | 0.5000 |
| 6 | 0.0000 | 1.8904 | 0.1032 | 1.9229 | 0.5000 |
| 8 | 0.0000 | 2.0937 | 0.1850 | 2.1360 | 0.5000 |
| 10 | 0.0000 | 2.1829 | 0.2726 | 2.2347 | 0.5000 |

Temperature increase with an increase in time at a different length. The velocity increases with an increase in time $(t)$ as the distance is varied.

$(T1), (T3)$ and $(T5)$ decreases with reducing with time. At a constant thermal conductivity, viscous dissipation parameter $(nu)$ controls the velocity and temperature at a different length of the flow.

The velocity at the lower plate rises from zero to maximum, where the velocity on the upper moving plate increases as the plate moves, thus leading to a convective acceleration.

(i) An increase in nu leads to an increase in temperature profile while a decrease in nu lowers the temperature profile.
(ii) The highest temperature recorded at the free stream region of the fluid flow corresponding to the area of maximum velocity.
(iii) For a constant $kappa\ = 0.1$, an increase in nu from 0-0.7 leads to a rise in temperature. Since the nu is increasing, the ratio of kinetic energy to thermal energy increases; hence, the temperature profile increases.

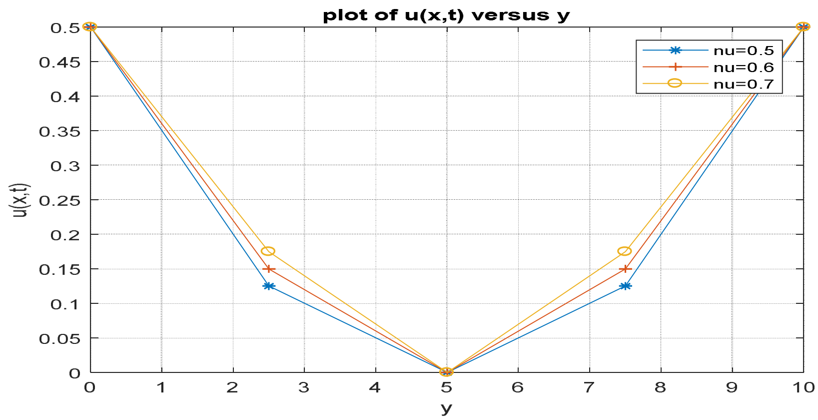


**Fig. 4. Velocity profile in x-direction at nu = 0:5 , nu = 0:6 nu = 0:7 and kappa = 0:1**

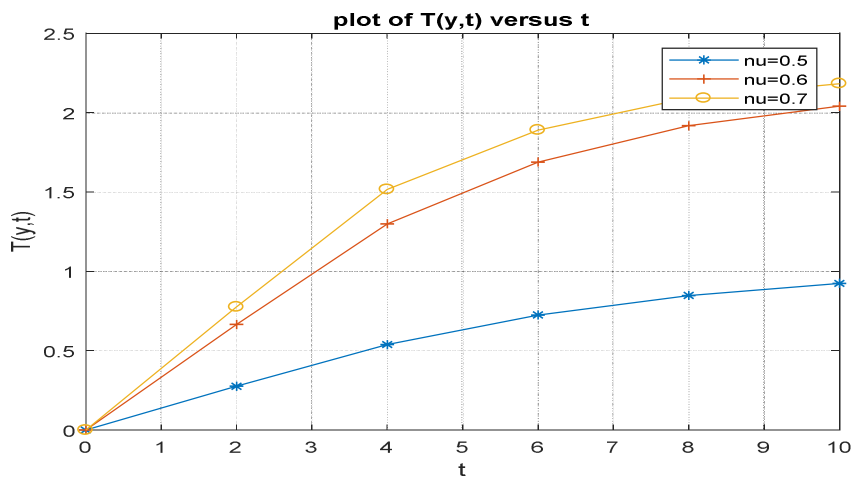


**Fig. 5. Temperature profile where nu = 0:5,nu = 0:6,nu = 0:7 and kappa = 0:1**

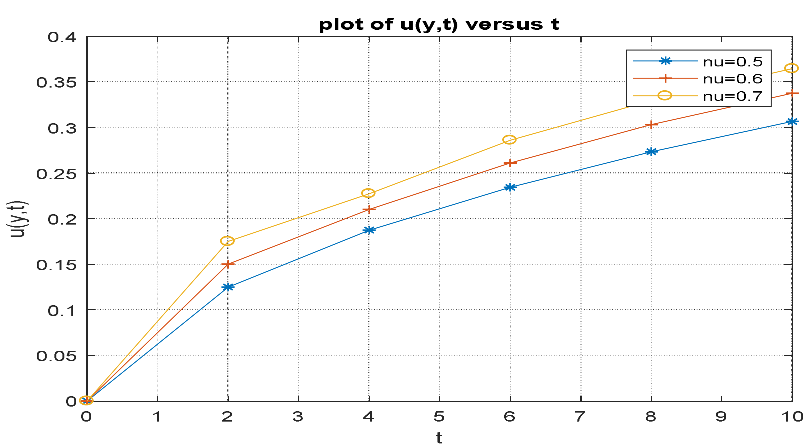


**Fig. 6. Velocity profile where nu = 0:5,nu = 0:6,nu = 0:7 and kappa = 0:1**

From Fig. 6, we can make the following observations; The velocity profile varies with nu. For short time intervals, an increase in the nu number leads to an increase in the velocity profiles. As the nu number increases from 0.5 to 0.7, the kinetic energy of the fluid increases, while the thermal energy between the layers of the fluid reduces, hence the velocity profiles increase. Viscous dissipation in MHD affects the temperature and velocity profile. From the tables above and Figs. 4, 5, and 6 viscous disposition parameters have great significance on MHD free convective fluid flow. The viscous term influences fluid properties, such as temperature, velocity, and time.

# 5 Conclusion

Viscous dispassion has a great significance on velocity and temperature profile in MHD fluid flow. At constant thermal conductivity, viscous dissipation increases with an increase in temperature and speed of the fluid and vice versa. The equations governing the hydromagnetic flow in our analysis were non-linear; hence, to obtain their solution, an efficient finite difference scheme was developed and employed. The results obtained for small changes in time indicate that the method was stable and convergent. In this study, the velocity and the temperature of the fluid in the boundary layer region remain constants.

Finally, the viscosity affected the velocity of the fluid. Different viscosities created when a fluid flows and the layers of the fluid moving past each other affected the resultant or overall velocity of moving fluid. The convective cooling of the plate controlled this increase, which results in the increased velocity of the fluid.

# Future Work

There is work underway to establish the effects of heat transmission in MHD flow with its subsequent viscosity.

# Competing Interests

Authors have declared that no competing interests exist.

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