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Stability Analysis in a Mathematical Model of Corruption in Kenya

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

The term corruption refers to the process that involves the abuse of a public trust or office for some private benefit. Corruption becomes a threat to national development and growth especially when there is no political will to fight it. Prevention and disengagement initiatives are part of EACC strategies used to fight corruption. Prevention strategies aim to stop or discourage citizens from engaging in corruption. Disengagement strategies attempt to reform corrupt individuals and to reclaim the stolen resources back to the public kitty. We describe prevention and disengagement strategies mathematically using an epidemiological compartment model. The prevention and disengagement strategies are modeled using model parameters. The population at risk of adopting corrupt ideology was divided into three compartments: S(t) is the susceptible class, C(t) is the Corrupted class, and M(t) is the corrupt political/sympathersizer class. The model exhibits a threshold dynamics characterised by the basic reproduction number $\mathcal{R}_0$. When $\mathcal{R}_0 < 1$ the system has a unique equilibrium point that is asymptotically stable. For $R_0 > 1$, the system has additional equilibrium point known as endemic, which is globally asymptotically stable. These results are established by applying lyapunov functions and the LaSalles invariance principle. Based on our model we assess strategies to counter corruption vice.

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1 Introduction

Like beauty, corruption is elusive to define, but incredibly easy to recognise. The term corruption is derived from the Latin word corruptus, which literally means to destroy [1]. It has been variously defined as an illegal act that involves the abuse of a public trust or office for some private benefit [2]. Transparency International defines corruption as misuse of entrusted power for private gain [3], while the World Bank considers it as an abuse of public authority for the purpose of acquiring personal gain [4]. Corruption assumes many diverse forms which vary from one society to the next. Nonetheless, the most conspicuous, and perhaps most familiar type of corruption in Kenya is the so called petty corruption, where a public official demands, or expects, speed money for doing an act which he or she is ordinarily required by law to do, or when a bribe is paid to obtain services which the official is prohibited from providing [5]. Another form of corruption is the grand corruption which occurs when a high-level bureaucrat who formulates government policy or is able to influence government decision-making, seeks, as a quid pro quo, payment, for exercising the extensive arbitrary powers vested in him or her [6].

Another form of corruption where Kenya lies is what is referred to as systemic corruption. This type of corruption is called entrenched corruption, this phenomenon occurs where corruption permeates the entire society to the point of being accepted as a means of conducting everyday transactions [7]. There are two powerful forces that stoke the fires of corruption: selfishness and greed. Because of selfishness, people turn a blind eye to the suffering their corruption inflicts, and rationalize it simply because they benefit from it. The more wealth they amass, the greedier they become for more [8]. Available research reveals that the causes of corruption are diverse and depend on the different contextual environments. One sociological study views corruption as being due to a divergence of attitudes and aims of different segments of society, or between the general public and the government [8]. The causes of corruption are economic, institutional, political or societal.

The economic causes of corruption are related to pecuniary considerations, representing corruption that is need-driven as opposed to greed driven. Whereas need driven corruption is intended to satisfy basic requirements for survival, corruption that is greed driven satisfies the desire for status and comfort which salaries cannot match. Institutional causes of corruption include monopoly and wide discretionary powers for public officers, poor accountability, lack of effective and efficient enforcement of the law, absence of institutional mechanisms to deal with corruption, existence of a weak civil society, and the absence of press freedom. Political corruption arises from the structure and functions of political institutions, and the acquisition and exercise of political power. Societal causes refer to the attitudes and practices of the community. When people are primarily motivated by personal, clan or other parochial loyalties rather than the rule of law, then conflicts of interest,cronyism and patronage reign supreme.

Corruption distorts markets and competition, breeds cynicism among citizens, undermines the rule of law, damages government legitimacy and erodes the legitimacy of the private sector [9]. A high level of corruption in a country reduces investment and the inflow of capital and results in a reduction in economic growth [5]. Corruption deepens poverty, it debases human rights; it degrades the environment; it derails development, including private sector development; it can drive conflict in and between nations; and it destroys confidence in democracy and the legitimacy of governments. It debases human dignity and is universally condemned by the worlds major faiths. Corruption represents a major threat to the rule of law, democracy and human rights, fairness and social justice, hinders economic development and endangers the proper and fair functioning of market economies [10].

It is with this in mind that this enquiry delves into the anti-corruption strategies that have so far been adopted by Kenya, in combating the vice, and postulates the reforms necessary for the effective management of corruption. There exists an abundance of literature on corruption: its general causes, effects and strategies on its management as discussed. Are these strategies working? or are the strategies directed to the wrong audience? or even the Kenya itself is not ready for this course?

The development of viable intervention strategies to mitigate corruption and corrupt practices requires a thorough understanding of the corrupt process, prevention and disengagement programs. Mathematical models can provide a first step in this direction. The use of differential equations to describe social science problems dates back, at least, to the work of Lewis F. Richardson [11] who pioneered the application of mathematical techniques by studying the causes of war, and the relationship between arms race and the eruption of war. Modern applications of compartmental models to the social sciences range from models of political party growth, to models of the spread of crime (see for instance) [12],[13],[14],[15],[16],[17], [18], [19], [20]. In recent years compartmental models have also been used to study terrorism, the spread of fanatic behavior, and radicalization [21], [22], [23], [24] [25],[26], [27]. In this paper we built a compartmental model. We introduced a political class that enabled us consider the role the corrupt political system plays in spread and prevention of corruption. We divided the population at risk of adopting corrupt ideology into three compartments, (S) Susceptible class consists of innocent individuals who are not corrupted and have no information on corrupt activities, (C) Corrupt class consists of individuals who take advantage of the office they hold to practice the vice and the general population already practicing corrupt activities at their lowest capacity and (M) Political class consists of individuals who hold political offices in the country and are using their political powers to engage in corrupt activities and sympathizers of the corrupt individuals (See Fig. 1). Using this model, we attempted to test the effectiveness of prevention and disengagement programs in countering and fighting corruption in Kenya.

We used the basic reproduction number $\mathcal{R}_0$ to evaluate strategies for countering and preventing corrupt activities. We showed that for $\mathcal{R}_0 < 1$ the system has one local asymptotically stable equilibrium where no Corrupt individual exists and for $\mathcal{R}_0 > 1$, the system has additional equilibrium where Corrupt vice is endemic to the population and latter equilibrium is asymptotically stable for $\mathcal{R}_0 > 1$. Thus, if $\mathcal{R}_0 < 1$ the corrupt vice will be eradicated,that is, Corrupt population will go to zero. When $\mathcal{R}_0 > 1$ the menace becomes endemic, that is, Corrupt individuals establish themselves in the population. The results we present in this paper are theoretical in nature and are fairly independent from the specific choices of the parameter values. Our model can, in principle, be used to evaluate the efficacy of preventive and disengagement programs in combating and fighting corruption whenever empirical data are known.

2 Corruption Model

2.1 Equations

A compartment model is used to describe the dynamics in this study. Let N(t) be the total population which is sub-divided into three sub-groups depending on the attributes for Corrupt activities as:

- (S) Susceptible
- (C) Corrupt
- (M) Political Corrupt

The rate at which susceptible are recruited to corrupt class is given by $\lambda S(t)$ that is, the force of recruitment. The transition rate of C(t) into M(t) is given by $\delta C(t)$ that is part of corrupt class are elected to hold political office despite of their integrity shortcoming. Recruitment rate of C(t) back into S(t) via secondary contacts (i.e., media and campaigning from opposing parties such as anti-corruption bodies, churches and government political will to fight this vice) is given by $\alpha C(t)$ while natural death rate is given by μ, the parameter β is the peer driven recruitment rate of S(t) into C(t) by Corrupt members and Sympathizers and τ is a factor by which the recruitment rate of S(t) into C(t) by political corrupt members exceeds the recruitment rate by individuals in C(t). We assume that there is homogenous mixing of the population and the effect of immigration and emigration are assumed not significant.

The transfer diagram for our system is as follows:

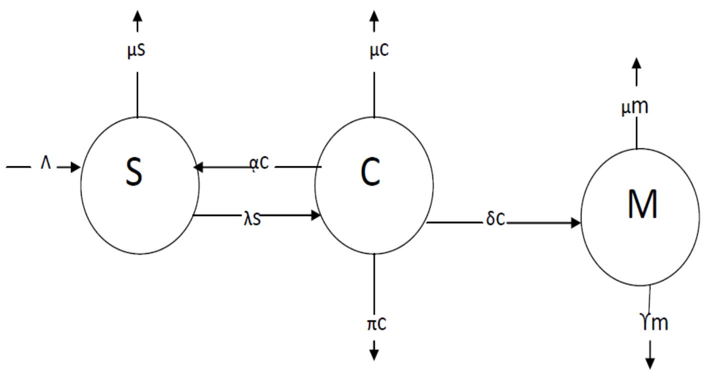

Fig. 1. The transfer diagram for the spread of corrupt ideology

From the flow chart above, we obtain the following system of equations representing the population dynamics of Corrupt addicts;

$$\begin{aligned}\frac{dS}{dt} &= \Lambda + \alpha C(t) - \lambda S(t) - \mu S(t)\\ \frac{dC}{dt} &= \lambda S(t) - \omega_1 C(t)\\ \frac{dM}{dt} &= \delta C(t) - \omega_2 M(t).\end{aligned} \tag{1}$$

Where $\omega_1 = \delta+\pi+\alpha+\mu$, $\omega_2 = \gamma+\mu$, there is a constant recruitment rate Λ into the susceptible and $N(t) = S(t) + C(t) + M(t)$. The starting conditions of the system of equations 1 were represented by $S(0) = S_0$, $C(0) = C_0$ and $M(0) = M_0$. The sum of system of equation 1 gives

$$\frac{dN}{dt} = \Lambda - \gamma M(t) - \pi C(t) - \mu N(t) \tag{2}$$

We begin our analysis by calculating equilibria for our model and determining conditions for their existence and stability.

3 Analysis of the Model

It is easy to show that the the feasible region Γ, given by $\Gamma = (S(t), C(t), M(t)) \in \Gamma \subset \mathcal{R}^3_+ : N(t) \leq \frac{\Lambda}{\mu}$ is a compact positive invariant and attracting set, it attracts all solutions of system of equations (1) with initial conditions in $\mathcal{R}^3_+$ .For the proof of similar statement see proposition 2.1 in [26]. Thus the model is epidemiologically well posed in the region Γ and can be analysed.

3.1 Corrupt free equilibrium (CFE) point

There is a unique Corrupt Free Equilibrium point (CFE) of system equations (1) with $S(t) = C(t) = T(t) = 0$ given by $E^0 = (S_0, 0, 0)$. Where $S_0 = \frac{\Lambda}{\mu}$.

3.2 The basic reproductive number $\mathcal{R}_0$

We denote by $\mathcal{R}_0$ the basic reproduction number which is the spectral radius of the next generation matrix calculated at at E^0 . $\mathcal{R}_0$ can be obtained as follows (see outlined procedure by Watmough et al [28] for more details).

Definition 3.2.1. The basic reproduction number $\mathcal{R}_0$ is the average number of Susceptible population, one Corrupted person can recruit in a purely susceptible population.

First we identify the infected(corrupt) compartments ,that in this case found to be C(t) and M(t) classes. In the immediate lemma the $\mathcal{R}_0$ is stated and a proof there after.

Lemma 3.2.1. Basic reproduction number of the System of equations (1) is given by

$$\mathcal{R}_0 = \frac{\beta S_0(\omega_2 + \tau\delta)}{\omega_1\omega_2} \tag{3}$$

Proof. Suppose $\mathcal{F}_c$ and $\mathcal{F}_m$ are the rate of arrival of newly Corrupt individual in compartment C and M, respectively. Let $V_i = \mathcal{V}^-{}_i - \mathcal{V}^+{}_i$, with $\mathcal{V}^+{}_i$ be the rate of transmission of individual into compartment $i(\in C, M)$ by all remaining methods, and $\mathcal{V}^-{}_i$ is the rate of removal of individuals from compartment associated with index i, where i is one of C and M. In this way, the matrices $\mathcal{F}$ and $\mathcal{V}$ associated with system of equations (1) are given by;

$$\mathcal{F} = \begin{bmatrix} \mathcal{F}_c \\ \mathcal{F}_m \end{bmatrix} = \begin{bmatrix} \lambda S(t) \\ 0 \end{bmatrix}, and, \mathcal{V} = \begin{bmatrix} \mathcal{V}_c \\ \mathcal{V}_m \end{bmatrix} = \begin{bmatrix} \omega_1 C(t) \\ -\delta C(t) + \omega_2 M(t) \end{bmatrix} \tag{4}$$

Consider the matrices F and V gotten by partially derivatives of C and M with time t

$$F = \begin{bmatrix} \frac{\partial \mathcal{F}_c}{\partial C} & \frac{\partial \mathcal{F}_c}{\partial M} \\ \frac{\partial \mathcal{F}_m}{\partial C} & \frac{\partial \mathcal{F}_m}{\partial M} \end{bmatrix} And, V = \begin{bmatrix} \frac{\partial \mathcal{V}_c}{\partial C} & \frac{\partial \mathcal{V}_c}{\partial M} \\ \frac{\partial \mathcal{V}_m}{\partial C} & \frac{\partial \mathcal{V}_m}{\partial M} \end{bmatrix} \tag{5}$$

Which in our situation takes the form

$$F = \begin{bmatrix} \beta S_0 & \beta\tau S_0 \\ 0 & 0 \end{bmatrix} And, V = \begin{bmatrix} \omega_1 & 0 \\ -\delta & \omega_2 \end{bmatrix} \tag{6}$$

Determining the inverse of matrix V yields;

$$V^{-1} = \frac{1}{\omega_1\omega_2} \begin{bmatrix} \omega_2 & 0 \\ \delta & \omega_1 \end{bmatrix} \tag{7}$$

Finally, we compute the next generation matrix of system of equations (1) given as $G = \mathcal{R}_0 = \rho\mathcal{F}\mathcal{V}^{-1}$;

$$G = \frac{1}{\omega_1\omega_2} \begin{bmatrix} \beta S_0 & \beta\tau S_0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_2 & 0 \\ \delta & \omega_1 \end{bmatrix} \tag{8}$$

The eigenvalues of matrix (8) are $\frac{\beta S_0(\omega_2+\tau\delta)}{\omega_1\omega_2}$ and 0. Since the matrix (8) has only one non-zero eigenvalue, it's a spectral radius ρ(dominant eigenvalue) denoted by $\rho\mathcal{F}\mathcal{V}^{-1}$ is:

$$\mathcal{R}_0 = \rho\mathcal{F}\mathcal{V}^{-1} = \frac{\beta S_0(\omega_2 + \tau\delta)}{\omega_1\omega_2} \tag{9}$$

□

The parameters are as described in section 2. The $\mathcal{R}_0$ is a very important parameter that quantifies the transmission of this corrupt ideology/activities in our model. From section 3.7, it is clear that $\mathcal{R}_0$ is a decreasing function of parameters γ, π and α. Note π and γ are parameters representing effectiveness on the fight of corruption by the constitutionally mandated bodies such as the anti-corruption commission, the judiciary, the parliament and the political will of the government in place. With a clear co-ordination between the three arms of government and the anti-corruption commission will probably mean an increase in these parameters. This, according to our model is a successful strategy to counter corrupt ideology since it will reduce $\mathcal{R}_0$. Another useful parameter is α, this parameter measures the level of corruption awareness. One of the mandate of EACC is to sensitize the public on corruption, typically these will be radio jingles, billboards, signs and more to sensitize people on the dangers of corruption. Based on this mandate, the public needs to know parameters that constitute corruption such as; bribery, abuse of office, fraud, embezzlement or misappropriation of public funds, breach of trust, an offence involving dishonesty in relation to taxes, bribery involving agents, secret inducements for advice, conflicts of interest, improper appointment and how these forms of corruption affect their day to day lives. This information will probably increase parameter α thereby reducing $\mathcal{R}_0$. More discussions are provided in section 3.8.

3.3 Local stability of corrupt free equilibrium (CFE) point

The signs of eigenvalues of the Jacobian matrix evaluated at corrupt free equilibrium were used to determine the necessary condition for local asymptotic stability of the corrupt free equilibrium point.

Definition 3.3.1. For local stability, perturbing the system slightly from the corrupt free equilibrium point, the system stays in the neighborhood of equilibrium point or, for asymptotic stability, it returns to equilibrium point. To show this, we state and proof theorem (3.3.1)

Theorem 3.3.1. *The CFE of the system of equations (1) is locally asymptotically stable whenever $R_0 < 1$ and unstable whenever $R_0 > 1$.*

Proof. From the system of equations (1), the Jacobian matrix evaluated at $E_0 = (S_0, 0, 0)$ the corrupt free equilibrium is given by

$$J_{E_o} = \begin{bmatrix} -\mu & \alpha - \beta S_0 & -\beta\tau S_0 \\ 0 & \beta S_0 - \omega_1 & \beta\tau S_0 \\ 0 & \delta & -\omega_2 \end{bmatrix} \tag{10}$$

The stability of this steady states can then be determined based on the signs of eigenvalues of the corresponding Jacobian matrix. We denote J_{E_0} as A_{22}.

$$A_{22} = \begin{bmatrix} -\mu & \alpha - \beta S_0 & -\beta\tau S_0 \\ 0 & \beta S_0 - \omega_1 & \beta\tau S_0 \\ 0 & \delta & -\omega_2 \end{bmatrix} \tag{11}$$

The determinant of matrix (11) is thus given by,

$$det A_{22} = -\mu\left(\omega_1\omega_2 - \beta\omega_2 S_0 - \delta\beta\tau S_0\right) \tag{12}$$

Simplifying equation (12) by introducing $\mathcal{R}_0$ results to;

$$detA_{22} = \mu\omega_1\omega_2 (\mathcal{R}_0 - 1) \tag{13}$$

If $\mathcal{R}_0 > 1$ then det $A_{22} > 0$. It follows that A_{22} and hence J_{E_o} has a positive eigenvalue. Therefore, E_0 is unstable if $\mathcal{R}_0 > 1$. Now, assume that $\mathcal{R}_0 < 1$, in this case det $A_{22} < 0$ and obviously trace $A_{22} < 0$ since $\omega_1 > \beta S$. Moreover, the second additive compound matrix [11] of A_{22} is

$$A_{22}^{(2)} = \begin{bmatrix} -\{\mu + (\omega_1 - \beta S)\} & \beta\tau S & \beta\tau S \\ \delta & -\{\mu + \omega_1\} & \alpha - \beta S \\ 0 & 0 & -\{(\omega_1 - \beta S) + \omega_2\} \end{bmatrix} \tag{14}$$

It follows from matrix [14] that $detA_{22}^{(2)} = \omega_1\omega_2\{(\omega_1 - \beta S) + \omega_2\}\{(\mathcal{R}_0 - 1) - (\frac{\mu^2 + \mu\omega_1 - \beta\mu S + \mu\omega_2}{\omega_1\omega_2})\}$, which is negative provide $\mathcal{R}_0 < 1$. By Lemma 3 [29], all eigenvalues of A_{22} have negative real parts and this is also true for J_{E_o}. Truly, E_0 is globally asymptotically stable if $\mathcal{R}_0 < 1$. Therefore, E_0 is locally asymptotically stable if $\mathcal{R}_0 < 1$. This completes the proof. □

3.4 Global stability of the corrupt free equilibrium (CFE) point

A lyapunov criterion for stability is applied to determine necessary conditions for global asymptotic stability of corrupt free equilibrium point.

Definition 3.4.1. An equilibrium point is global (asymptotically) stable if it is unique equilibrium of the dynamical system and the property hold globally (its domain of attraction is entire state space)[24].

Theorem 3.4.1. $\mathcal{R}_0 < 1$, *then the corrupt free equilibrium point* $E_0 = (S_0, 0, 0)$ *of system of equations (1) is globally asymptotically stable in* Γ *and unstable if* $\mathcal{R}_0 > 1$

Proof. We propose the following Lyapunov function for system of equations (1)

$L(S, C, M) = \left(S - S_0 - S_0 ln\left(\frac{S}{S_0}\right)\right) + \alpha_1 C + \alpha_2 M$

The α_1 and α_2 are positive constants determined at Corrupt Free Equilibrium point. If the lyapunov function $L(S, C, M)$ satisfies the conditions, $L(S_0, C_0, M_0) = 0$ and $L(S_0, C_0, M_0) > 0$, then it is a positive definite. If $\frac{dL(S^0, C^0, M^0)}{dt} = 0$ and $\frac{dL(S,C,M)}{dt} < 0$ then $\frac{dL(S,C,M)}{dt}$ is a negative definite. Determining the time derivative of the lyapunov function L along the trajectories of system of equations (1);

$$\frac{dL(S, C, M)}{dt} = \left[1 - \left(\frac{S_O}{S}\right)\right]\frac{dS}{dt} + \alpha_1\frac{dC}{dt} + \alpha_2\frac{dM}{dt} \tag{15}$$

Substituting in $\frac{dS}{dt}, \frac{dC}{dt}$ and $\frac{dM}{dt}$ results to,
$\frac{dL(S,C,M)}{dt} = \left(1 - \frac{S_0}{S}\right)(\mu S_0 + \alpha C - \lambda S - \mu S) + x_1(\lambda S - \omega_1) + x_2(\delta C - \omega_2)$

Expanding and simplifying result to,

$\frac{dL(S,C,M)}{dt} = \left(1 - \frac{S_0}{S}\right)(\mu S_0 - \mu S) + \alpha C + \lambda S - \alpha C\left(\frac{S_0}{S}\right) - \lambda S_0 + x_1\lambda S - x_1\omega_1 C + x_2\delta C + (-x_2\omega_2 - \beta\tau S_0) M$
$= -\mu\frac{(S - S_0)^2}{S} - \alpha C\left(\frac{S_0}{S}\right) + (1 + x_1)\lambda S + (\alpha - \beta S_0 - x_1\omega_1 + x_2\delta) C + (-x_2\omega_2 - \beta\tau S_0) M$

setting λS and C to zero, results;

$x_1 + 1 = 0$, $x_1 = -1$
$\alpha - \beta S_0 + \omega_1 + x_2\delta = 0$, $x_2 = \frac{\beta S_0 - \alpha - \omega_1}{\delta}$

With introduction of $\mathcal{R}_0$ the derivative of the lyapunov yields;

$$\frac{dL(S, C, M)}{dt} = -\mu\frac{(S - S_0)^2}{S} - \alpha C\left(\frac{S_0}{S}\right) + (\omega_1\omega_2(\mathcal{R}_0 - 1) + \alpha\omega_2) M \tag{16}$$

For $\frac{dL(S,C,M)}{dt} < 0$ then the necessary and sufficient condition is $(\omega_1\omega_2(\mathcal{R}_0 - 1) + \alpha\omega_2) < 0$ that is $\mathcal{R}_0 < 1 - \frac{\alpha}{\omega_1}$. By inspection $\left(\frac{\alpha}{\omega_1} \geq 0\right)$ hence the $\mathcal{R}_0 < 1$ is also a sufficient condition. When $\mathcal{R}_0 < 1$, it is not difficult to verify that the largest invariant set of system of equations (1) on the set $\left((S, C, M) \in \Gamma : \frac{dL}{dt} = 0\right)$ is a singleton (E_0). From LaSalles Invariance Principle [30], the Corrupt free equilibrium (E_0) is therefore globally stable on the set Γ given that $\mathcal{R}_0 < 1$. □

3.5 Existence of the corrupt persistence equilibrium (CPE) point

The condition necessary for force of recruitment to be positive was determined to predict the presence of corrupt persistence. If $E^*(S^*, C^*, M^*)$ is endemic equilibrium state, then $E^*(S^*, C^*, M^*) \neq (0, 0, 0)$.

Theorem 3.5.1. *A positive Corrupt persistent equilibrium point exists and is locally asymptotically stable whenever $\mathcal{R}_0 > 1$*

Proof. In order to obtain the Corrupt persistent equilibrium state, we solve equations (17, 18, 19) simultaneously;

$$\Lambda + \alpha C - \lambda S - \mu S = 0 \tag{17}$$

$$\lambda S - \omega_1 C = 0 \tag{18}$$

$$\delta C - \omega_2 M = 0. \tag{19}$$

let the corrupt persistent equilibrium point be $E^* = (S^*, C^*, M^*)$ of system of equations 1 and the force of recruitment into corrupt ideology be given by $\lambda^* = \beta(C^* + \tau M^*)$. Solving the system of equations (1) in terms of λ^* and $C, M \neq 0$ we obtain;

$$S^* = \frac{\Lambda\omega_1}{\omega_1\lambda^* + \omega_1\mu - \alpha\lambda^*} \tag{20}$$

$$C^* = \frac{\lambda^*\Lambda}{(\omega_1\lambda^* + \omega_1\mu - \alpha\lambda^*)} \tag{21}$$

$$M^* = \frac{\lambda^*\delta\Lambda}{\omega_2(\omega_1\lambda^* + \omega_1\mu - \alpha\lambda^*)} \tag{22}$$

substituting C^* and M^* into $\lambda^* = \beta(C^* + \tau M^*)$ and factoring in $\mathcal{R}_0$ we obtain;

$$\lambda^* = \frac{\mu\omega_1}{(\omega_1 - \alpha)}(\mathcal{R}_0 - 1) \tag{23}$$

It therefore follows, that the condition necessary for $\lambda^* > 0$ is whenever $\mathcal{R}_0 > 1$. The expression for corrupt persistent equilibrium point is evaluated by substituting equation (23) into equations (20), (21) and (22) to yield;

$S^* = \frac{S}{\mathcal{R}_0}$
$C^* = \frac{\mu(\mathcal{R}_0-1)S^*}{(\omega_1-\alpha)\mathcal{R}_0}$
$M = \frac{\mu^2\omega_1(\mathcal{R}_0-1)\delta S_0}{\mu\omega_2(\omega_1-\alpha)(\mathcal{R}_0-1)+\omega_1\mu}$

Therefore the condition necessary and sufficient for $\lambda > 0$ is $\mathcal{R}_0 > 1$ which completes our proof. □

3.6 Global stability of corrupt persistence equilibrium (CPE) point

To show the global stability of the corrupt persistence equilibrium we state and prove the immediate theorem,

Theorem 3.6.1 *If $\mathcal{R}_0 > 1$, then the endemic equilibrium E^* of system of equations (1) is globally asymptotically stable and unstable otherwise.*

Proof. To proof this, we propose the following Lyapunov function;

$$P \quad = \quad S^* g(x) ln\left(\frac{S}{S^*}\right) + \alpha_1 C^* g(x) ln\left(\frac{C}{C^*}\right) + \alpha_2 g(x) M^* ln\left(\frac{M}{M^*}\right) \tag{24}$$

$g(x)$ is a function of x given as $g(x) = x - 1 - lnx$. It's obvious that P is C^1, $P(x^*) = 0$, and $P > 0$ for any $q \in R$ such that $q \neq x^*$.

Getting the derivative of P along solutions of system of equations (1) yields,

$$P' \quad = \quad \left(1 - \frac{S^*}{S}\right) S' + \alpha_1 \left(1 - \frac{C^*}{C}\right) C' + \alpha_2 \left(1 - \frac{M^*}{M}\right) M' \tag{25}$$

Factoring in S', C' and M',

$P' = \left(1 - \frac{S^*}{S}\right) [\Lambda - \alpha C - \beta (C + \tau M) S - \mu S] + \alpha_1 \left(1 - \frac{C^*}{C}\right) [\beta (C + \tau M) S - \omega_1 C]$
$+ \alpha_2 \left(1 - \frac{M^*}{M}\right) [\delta C - \omega_2 M]$

Upon simplification gives;

$P' = \Lambda + \alpha C - \beta CS - \beta\tau MS - \mu S - \Lambda\left(\frac{S^*}{S}\right) - \alpha C\left(\frac{S^*}{S}\right) + \beta CS^* + \beta\tau MS^* - \mu S^* + \alpha_1\beta CS + \alpha\beta\tau MS - \alpha_1\omega_1 C - \alpha_1\beta C^* S - \alpha\beta\tau MS\left(\frac{C^*}{C}\right) + \alpha_1\omega_1 C^* + \alpha_2\delta C - \alpha_2\omega_2 M - \alpha_2\delta C(t)\left(\frac{M^*}{M}\right) + \alpha_2\omega_2 M^*$

Where $C = (\Lambda - \mu S^* + \alpha_1\omega_1 C^* + \alpha_2\omega_2 M^*)$,

For simplicity, denote $w = \left(\frac{S}{S^*}\right)$, $x = \left(\frac{C}{C^*}\right)$,$y = \left(\frac{M}{M^*}\right)$

$P' = (\Lambda - \mu S^* + \alpha_1\omega_1 C^* + \alpha_2\omega_2 M^*) - (\alpha C^* + \beta C^* S^* - \alpha_1\omega_1 C^* + \alpha_2\delta C^*)\, x$
$+ (-\beta C^* S^* + \alpha_1\beta C^* S^*)\, wx + (-\beta\tau S^* M^*)\, wy - (\mu S^* + \alpha_1\beta S^* C^*)\, w + \Lambda\left(\frac{1}{w}\right) - \alpha C^*\left(\frac{x}{w}\right)$
$+ (\beta\tau M^* S^* - \alpha_2\omega_2 M^*)\, y - \alpha_1\beta\tau S^* M^*\left(\frac{wy}{x}\right) - \alpha_2\delta C^*\left(\frac{x}{y}\right)$

We then choose $a_i > 0 (i = 1, 2)$ which are constants suitable for function $A(w, y, x)$ to be negative definite or semi-definite. Function $A(w, x, y)$ is rewritten with constants $a_i > 0(i = 1, 2)$ as the following form

$$-\sum_{k=1}^{k} a_k (L_{k,1} + L_{k,2} + ... + L_{k.n_k} - n_k), \tag{26}$$

where $(a \geq 0)$ and $(k = 1, 2, 3......k)$, $(L_{k.i})$ is an expression only including multiplication and division of(w, x, y) and,

$$\prod_{i=1}^{n_k} L_{k.i} = 1. \tag{27}$$

As in [24] and [31], define a set D of the above terms by

$\left(w, x, y, wx, wy, \frac{1}{w}, \frac{x}{w}, \frac{x}{y}, \frac{wy}{x}\right)$

Then we can consider the arithmetic means and geometric means of the following two sets to determine a_1 and a_2

$\left(w.\frac{1}{w}\right), \left(\frac{1}{w}.\frac{wy}{x}.\frac{x}{y}\right)$.

Referring to the method used in [31] we construct a lyapunov function as linear combination of the terms above:

$$H(w,x,y) = a_1\left(2 - w - \frac{1}{w}\right) + a_2\left(3 - \frac{x}{y} - \frac{wy}{x} - \frac{1}{w}\right) \tag{28}$$

Where the coefficients $a_1 \geq 0$ and $a_2 \geq 0$ are left unspecified. We therefore want to determine suitable parameters $\alpha_1 > 0$ and $\alpha_2 > 0$ such that $A(w,x,y) = H(w,x,y)$. Equating the coefficients terms in A(w,x,y) and H(w,x,y) with the terms (wx), (x) and(y) of function A(w,x,y) not appearing in function H(w,x,y), their coefficients are equated to zero, which gives the followings equations:

$$\begin{aligned} \alpha C^* + \beta S^* C^* - \alpha_1\omega_1 C^* + \alpha_2\delta C^* &= 0 \\ -\beta S^* C^* + \alpha_1\beta S^* C^* &= 0 \\ -\beta\tau S^* M^* + \alpha_1\beta\tau S^* M^* &= 0 \\ \beta\tau S^* M^* - \alpha_2\omega_2 M^* &= 0 \\ -\alpha C^* &= 0 \end{aligned} \tag{29}$$

It follows from equation (29) that α_1 and α_2 can be uniquely determined as;

$$\begin{aligned} \alpha_1 &= 1 \\ \alpha_2 &= \frac{\beta\tau S^*}{\omega_2} \end{aligned} \tag{30}$$

Consequently, the Lyapunov function (24) is specified. Function A(w,x,y) is in turn given as,

$$\begin{aligned} P' &= (\Lambda - \mu S^* + \alpha_1\omega_1 C^* + \alpha_2\omega_2 M^*) - (\alpha C^* + \beta C^* S^* - \alpha_1\omega_1 C^* + \alpha_2\delta C^*)\,x \\ &+ (-\beta C^* S^* + \alpha_1\beta C^* S^*)\,wx + (-\beta\tau S^* M^*)\,wy - (\mu S^* + \alpha_1\beta S^* C^*)\,w + \Lambda\left(\tfrac{1}{w}\right) - \alpha C^*\left(\tfrac{x}{w}\right) \\ &+ (\beta\tau M^* S^* - \alpha_2\omega_2 M^*)\,y - \alpha_1\beta\tau S^* M^*\left(\tfrac{wy}{x}\right) - \alpha_2\delta C^*\left(\tfrac{x}{y}\right) \triangleq A(w,x,y) \end{aligned}$$

Letting A(w,x,y) = H(w,x,y) and comparing like terms results to,

$$\begin{aligned} \mu S^* + \alpha_1\beta S^* C &= a_1 \\ \Lambda &= a_1 + a_2 \\ \alpha_1\beta\tau S^* M^* &= a_2 \\ \alpha_2\delta C^* &= a_2 \end{aligned} \tag{31}$$

From equation (31) it follows that;

$$\begin{aligned} a_1 &= S^*(\mu + \beta C^*) \\ a_2 &= \beta\tau S^* M^* \end{aligned} \tag{32}$$

So function $H(w,x,y)$ is also uniquely determined, the derivative of the Lyapunov function is given by

$$\frac{dP}{dt} = a_1\left(2 - w - \frac{1}{w}\right) + a_2\left(3 - \frac{x}{y} - \frac{wy}{x} - \frac{1}{w}\right) \leq 0 \tag{33}$$

where $a_1, a_2 \geq 0$ are as given in equation (32). By the arithmetic and the associated geometric means, we have $\frac{dP}{dt} \leq 0$ and the equality holds if and only if $(w = x = y = 1)$, that is, $\left(S = S^*, \frac{C}{C^*} = \frac{M}{M^*}\right)$. It can be easily verified that the largest invariant set of system of equations (1) on the set

$\left(\Gamma = (S, C, M) \in R_+^3 : \frac{dP}{dt} = 0\right) = (S, C, M) \in R^3 : S = S^* = \frac{C}{C^*} = \frac{M}{M^*}\right)$

is the singleton E^*. We claim that the largest invariant set in Γ consist of endemic equilibrium E^*. By LaSalle's invariance principle [32], we concluded that all solutions of system of equations (1) that start in R^3 limits to E^*. In fact, let $(S(t), C(t), M(t))$ be such solution in Γ, Denote $\frac{C}{C^*} = \frac{M}{M^*}$.Then,

$$0 = \frac{dS}{dt} = \frac{dS^*}{dt} = \Lambda + \alpha C - \beta\left(C^* + \tau M^*\right) S^* - \mu S \tag{34}$$

Which implies that,

$$C = \frac{\Lambda + \alpha C - \mu S}{\beta\left(C^* + \tau M^*\right) S^*} = 1 \tag{35}$$

Clearly $(S, C, M = E^*)$. This is consistence with our claim. Thus, by the LaSalles Invariance Principle [30], the endemic equilibrium $(S, C, M = E^*)$ is globally asymptotically stable on the set Γ when $\mathcal{R}_0 > 1$. □

3.7 Analytical sensitivity analysis of the basic reproduction number and their social interpretation

Partial differentiation of the basic reproduction number with respect to the model parameter is carried out. Later social interpretation is then provided in the later section of discussion. Partial derivative of $\mathcal{R}_0$ with relative to model parameters β, γ, τ, δ, π and α give;
$\frac{\partial \mathcal{R}_0}{\partial \beta} = \frac{S_0(\gamma+\mu+\tau\delta)}{(\delta+\pi+\alpha+\mu)(\gamma+\mu)} > 0$, $\frac{\partial \mathcal{R}_0}{\partial \gamma} = \{\frac{\beta S_0}{\omega_1\omega_2} - \frac{1}{\omega_1^2\omega_2}\} < 0$, $\frac{\partial \mathcal{R}_0}{\partial \tau} = \frac{\beta S_0 \delta}{\pi\omega_1\omega_2} > 0$, $\frac{\partial \mathcal{R}_0}{\partial \delta} = \frac{\beta S_0 \tau}{(\omega_1+\omega_2)} > 0$, $\frac{\partial \mathcal{R}_0}{\partial \pi} = -\frac{\beta S_0(\omega_1+\tau\delta)}{\omega_1^2\omega_2} < 0$ and $\frac{\partial \mathcal{R}_0}{\partial \alpha} = -\frac{\beta S_0(\omega_2+\tau\delta)}{\omega_1^2\omega_2} < 0$

The partial derivative of $\mathcal{R}_0$ with model parameters that give values less than 0 are inversely proportional to $\mathcal{R}_0$. Increasing these parameters is a successful strategy in fighting corruption in Kenya.

4 Discussion

It is believed that this study will positively contribute to efforts by the Kenyan government and to comprehensively address the high prevalence of corruption in the country. The $\mathcal{R}_0$ is directly affected by the parameters β, τ and δ which measure the level of interaction between corrupt and susceptible and transition of corrupt individuals to political positions respectively. For instance, parameter β takes care of events such as at roadblock driving license with some money inside is handed to traffic police in presence of quiet passengers to evade the rule of law. These passengers are being recruited into corrupt activities by the police and the drivers of the vehicles, and possibly upon owning their own vehicles they will do the same. Similarly parameter δ accommodates events such as electing corrupt, dishonest, ineffective and incapable individuals to leadership positions possibly because they are tribe mates, friends, relatives or by receiving bribes symbolizes that the electorate and their leaders are corrupt. Cheating in examinations, illegal businesses such as tax evasion and poaching, aiding in cheating, bribes to secure a particular favor and more are acts of corruption. The parameters β, δ and τ take care of such forces of recruitment.

The point is that controlling corruption requires a multifaceted approach that addresses the numerous causes, components and structural issues that corruption entails. First, it is clear that, to reduce these parameters linked directly to $\mathcal{R}_0$ calls for active participation and long term commitment by a variety of anti-corruption actors such as governments, civil society, media, academicians, the private sector and international organisations. Secondly, there must be genuine political goodwill from the entire political landscape including the electorate and a strong legislative base that empowers the concerned bodies to investigate and prosecute corruption cases, and recover the proceeds of crime. Thirdly, a clear, and well coordinated strategy that encompasses effective enforcement of laws, prevention of corruption by eliminating from systems the opportunities for corruption and adequate provision of both human and financial resources. Educating the public about corruption and its dangers, enlisting their support in fighting the vice and involvement of the people as part and parcel of the strategy with an understanding from the onset that they have a personal responsibility to fight corruption themselves, for their own good and for the good of posterity of their nation. Finally, a realization that beating corruption takes time and that once corruption has been brought under control, it must be kept from escalating by enuring that the threshold in place is maintained. This paper goes to confirm several reports on corruption assessment conducted by EACC and Research Institutions which reveal that fight against corruption is a collective responsibility and personal initiative.

We have seen that $\mathcal{R}_0$ is expressed in terms of model parameters, therefore it was easy to evaluate numerous strategies to curb this menace. There were a number of shortcomings for instance it was difficult to obtain empirical data regarding corruption prevention strategies. Therefore more studies should be done to determine more reliable quantifiable results and perform numerical simulations to confirm these theoretical results.

5 Conclusion

Pervasive corruption has slowed growth and deepened the poverty levels in the country. Eliminating corruption will free significant resources for investment in infrastructure and in programmes that deliver services to the poor. Corruption continues to pose one of the greatest challenges facing Kenya. It continues to undermine good governance and distorts public policy, leading to misallocation of resources. It has contributed to slow economic growth as well as discouraged and frustrated both local and foreign investors. The poor management, excessive discretion in government, appointments of people of dubious characters and political interference and lack of respect for professionalism has led to widespread corruption, gross abuse of public office in many government departments and incorrigible tolerance. For these reasons the solution of the current national crisis is to be found in our ability to reclaim professionalism and confidence in public officers, and guaranteeing efficiency.

6 Recommendation

The constitutional reform which was done in 2010 must be accompanied by the promotion of a culture of constitutionalism that embraces key concepts of democracy, good governance, transparency and accountability. Efforts should therefore be made to grant Anti-corruption commission prosecutorial powers. For the war on corruption to succeed in Kenya, it needs the consistent drive and determination of all stakeholders. Integrity is a culture, and as such must be embedded in every facet of society. These changes will require sustained pressure from below and leadership from above. Civil society and the media must continue to play a watchdog role to subject government to the light of intense public scrutiny. Not forgetting that for all these recommendation to go along way, there is a need of genuine political will and transformational leadership at all levels of the political landscape, from the electorate to government. Leaders must display conviction, emphasize the importance of purpose and commitment, challenge followers with high standards, and provide encouragement and meaning for what needs to be done. The media should break this information concerning corruption in digestible state for the lowest level population consumption. This is possible because nearly every tribe in kenya has a radio or television broadcasting in their native language. Lastly, what a terrorist and a drug dealer do is exactly what a corrupt person do, to achieve a corrupt free society, then corruption should be fought with magnitude similar to that of terror and drug dealings. Without this, government and anti-corruption statements on reforming the public service, strengthening transparency and accountability, and doing away with corruption, will remain a mere rhetoric.

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Competing Interests

Authors have declared that no competing interests exist.

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