A Slacks-based Measure in the Presence of Ratio Variable with Convexity Consideration

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

The nature of input-output relationships in general and ratio data in particular has important consequences for practitioners when the data envelopment analysis method is used to measure technical efficiency of decision making units or production units. Since the data envelopment analysis approach was introduced several studies tried to develop the model from different aspects including when the model deals with ratio data. To date, none of these studies was able to address the aforementioned problem properly and as a result most of them suffered from a lack of clarity in the presence of input-and-output ratios. This study proposes a slacks-based measure of efficiency in the presence of ratio variable. Our approach deals directly with the input excess and the output shortfalls of the decision making units' concerns, and as a result, improved measuring efficiency scores.

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1 Introduction

The use of data envelopment analysis in measuring the relative efficiency score of production unit is rapidly increasing [1,2,3]. Prior to using this technique, the popular methods to compute the relative efficiency score of a DMU through estimating a frontier production or cost functions were two-fold: parametric and non-parametric. Parametric frontiers rely on the specification of a particular functional form for production, while non-parametric frontiers have the advantage of not being limited by a priori functional forms. Parametric and nonparametric frontiers also differ from each other on the ground of their error distributions (e.g., half-normal, truncated normal, exponential, gamma, etc.). In addition, to use parametric frontier estimation it is required to pre-specify functional forms (e.g., Cobb-Douglas, translog, transcendental, etc.). However, nonparametric estimators do not rely on these assumptions. Moreover, frontiers are either deterministic or stochastic (non-deterministic). In the former, any deviation from the frontier is assumed to be due to inefficiency. In the latter model, a compound error term is defined to account for inefficiency as well as random noise. A nonparametric estimator is a robust estimator that allows the data to determine the shape of the functional form without any constraints derived from relevant economic theory. Interested readers can find more about the application of estimating parametric and nonparametric frontiers in [4,5,6,7,8].

Data envelopment analysis (DEA) is a nonparametric technique that encompasses data in a geometrically way. According to Law et al. [9], a producer is said to be technically efficient if a firm is using the minimum amount of inputs to produce a maximum level of output. The DEA solves for an artificial frontier comprising a linear combination of the most technically efficient units in the sample. The efficiency scores for a given DMU are calculated relative to this efficient frontier. We can also define DEA scores as the ratio of the weighted average of outputs over the weighted average of inputs when the scores are optimized. The optimization technique compares a couple series of outputs and inputs values. The first series is called the observed values which are simply the particular data points. The second series of values is called the optimal values which are accounted from the best performing DMUs. Since such an optimization is similar to a linear programming method, the constraints in the DEA technique ensure that the weights cannot yield a ratio of outputs to inputs greater than one, implying that a score of unity is the score of efficient units in the sample. Opponents of using DEA technique in applied economics argue that the results are suffered from lack of any statistical analyses, whereas the proponents of using this technique emphasize on using all the information when it calculates the optimal weights of efficiency scores for each DMU [9]. Moreover, Charnes et al. [10] stated that the DEA technique is readily configured to a multiple-input and/or multiple-output framework, rendering it more useful than cost analysis depending on the data available. It is a general consensus that the DEA technique is a proper method to measure DMUs' technically efficiency scores in those applied studies when the presence of public policy and/or regulatory oversight is imperative [4,5,9].

The use of DEA technique, however, is not without major drawbacks. For example, most studies that employed the DEA approach undertook with absolute numerical data, which among other things reflect the size of the units of observation [11]. In other studies, researchers used ratio variable rather than absolute numbers an input (input-ratios) and/or output (output-ratio). This is imperative as it accurately reflects the underlying production function and/or facilitates comparison between units. For instance, in efficiency measurement of financial institutions that financial ratio is included in the model as output variable [12]. Precedent studies expressed that when input and/or output includes a ratio variable then the Banker, Charnes, and Cooper [13] model should be used instead of the Charnes, Cooper, and Rhodes' [14] model [15]. For instance, Emrouznejad and Amin [12] raised the convexity problem of using standard DEA models in a production possibility frontier at the presence of input-rations and/or output-ratio. The researchers specified a new convexity assumption when data contains a ratio variable and proposed a series of modified DEA models which rectify the aforementioned problem. The main objective of this study is to present a set of modified slacks-based measure (SBM) models that take into account the correct convexity of DMUs when the models encompasses a ratio variable and discussed the properties of the proposed models.

The rest of paper is as follows. Section 2 reviews recent studies in different industries that used the SBM models as part of their methodologies. Section 3 presents the methodological framework, discusses the proposed previous solutions for DEA models when a ratio variable is included in the assessment model, and introduces some new SBM models to rectify the problem in the presence of ratio data. The penultimate section presents a numerical example and compares the findings of the model between modified SBM models and BCC models. Section 5 concludes some remarkable points.

2 Recent Studies on Slacks-based Measure (SBM) Models

Literature shows a reasonable number of publications on SBM models despite they have recently been developed. In this section we briefly review some of those studies and refer interested readers to find more.

In a study, Bian et al. [16] expressed that China's scale-driven economic development has led to great energy consumption in the production process during the recent 30 years. From 1980 to 2012, China's GDP has increased about 20.88 times, while energy consumption (standard coal equivalent, SCE) in 2012 is about 6.0 times that of 1980. The aggressive increase of energy consumption has also given rise to energy shortages, energy crisis, energy price going up and serious pollution and ecological problems. For instance, in 2010, CO_2 emissions in China accounted for about 28.53 percent of the world's total CO_2 emissions, which was much larger than that of the U.S. (i.e., 15.88%). In this regard, China has also been the largest emitter of CO_2 emissions in the world. With the consideration of economic growth, environmental pressure and sustainable development, energy use has become a major concerned research issue in recent years. Bian et al. [16] identified that the economic system in China composed of three internal parallel industries: primary, secondary and tertiary industries. The researchers examined energy efficiency of the economic system in China and its industries using a parallel slacks-based DEA measure approach during 1986–2012. The findings of the study were three-fold: (*i*) their slacks-based DEA measure model was better than the standard DEA model in providing a viable efficiency measurement for the Chinese energy industry which enabled the researchers to identify sources of inefficiencies caused by internal industries, (*ii*) Bian et al. [16] recognized that the weak energy performance of the secondary industry was the major source of the inefficiencies in country's economic system, (*iii*) the economic system observed improvement in energy efficiency performance during the study time period with an exception during 2001–2005, which helped the industry to manage its energy consumption after all.

Production stage and pollutant abatement stage are the two internal stages in the process of a regional industrial system in China [17]. The efficiencies of Chinese regional industrial systems were examined in a research conducted by Bian et al. [17] who used a two-stage slacks-based measure approach in the data envelopment analysis framework. One of the advantages of this model is to decompose total efficiency of the whole regional industrial system into the aforementioned stages simultaneously. The result of the study showed substantial disparities in regional industrial systems' efficiencies caused mostly by the abatement stage.

It is a general consensus that one of the sources of pollution in cities near ports is the air pollutant emissions [18]. Some examples of its adverse effects are depleting the ozone layer, increasing the green-house effect, and producing acid rain. In a study Lee et al. [18] assessed port's environmental performance by examining the relationship between port and city functions through emerging issues like environmental influences. The researchers used a slacks-based data envelopment analysis model to estimate the environmental efficiency of port cities. They chose a couple of desirable output variables (i.e., gross domestic regional product and container throughput), three undesirable output variables (i.e., nitrogen oxide, sulfur oxide, and carbon dioxide emissions), and labor population as their explanatory variable of the model. The researchers were able to identify Singapore, Busan, Rotterdam, Kaohsiung, Antwerp, and New York as the most efficient port cities from environmental perspective, whereas Tianjin as the least environmentally efficient port city.

Akther et al. [19] used a slacks-based inefficiency measure and the directional technology distance function to examine the performance of 21 commercial banks (i.e., 19 private and 2 public) in Bangladesh during 2005-2008. One view of finance discusses the inherent role of credits in developing the rural and urban

economies. These credits could be in the format of mobile capital that is used in any projects with high net present values. The researchers extended the model of Fukuyama and Weber [20] to measure performance by assuming a black-box production structure that was opened and assessed by using a two stage network production structure [21]. In the first stage, Akther et al. [19] noticed that deposits were used by commercial banks as intermediate output through combining labor and capital as inputs. In the second stage, the deposits were included in commercial banks' asset portfolios including loans and securities investments. The researchers also considered non-performing loans caused by sudden changes in the values of assets, the status of the economy, and normal functioning of the banking systems. The result showed that deposits were under-produced annually, on average, in the first stage of production, which led to approving fewer loan applications, reducing the volume of securities investments, and yielding substantial default loans (i.e., up to 4 percent of total assets). Had the bank managers operated efficiently the percentage of default loans could have reduced to 2.9 to 3.5 percent. As a result of such reduction commercial banks were able to release their financial sources, expand new loans, and enhance securities portfolios in the country.

3 Methodological Framework

3.1 Problem with ratio data

In a study, Emrouznejad and Amin [12] showed that convexity assumption may fail when at least one of the input or output variable is ratio, and demonstrated that using the standard DEA models for the observation containing ratio data may result incorrect efficiency scores. The researchers provided a couple of solutions for a situation where input and/or output contains a ratio variable which is as follows. First, (*i*) if the input is being ratio variable, the recommendation is to use both the numerator and denominator of any input-ratio variable and place them accordingly as an additional input and output into the model, and (*ii*) if the output is being ratio variable, the recommendation is to use both the numerator and denominator of any output-ratio variable and place them accordingly as an additional output and input into the model. Second, instead of using the standard convexity combination for ratio variable the researchers suggested to use the correct convexity combination of ratio variable which is defined as the ratio of convex combination of numerators to the convex combination of denominators [12].

3.2 The slacks-based measure (SBM) model

To circumvent the inclusion of output-ratio and/or input-ratios) problems, Tone [22] proposed a slacks-based measure (SBM) model that is unit free and monotone which makes its efficiency evaluation irrespective of the units of the measurements used for different output and inputs. The SBM model directly uses slack variables, i.e., input excesses and output shortfalls of DMUs. A short explanation of the model is as follows:

Assuming that there are *n DMUs*, each DMU_j $(j = 1.2. \dots. n)$ is producing *s* positive output y_{rj} $(r = 1.2. \dots. s$ using *m* non-negative inputs x_{ij} *i=1.2. ….m.* In the following the SBM model that is based on the production possibility set of the BCC model to measure efficiency of a DMU is presented given the condition that

$$\sum_{j=1}^{n} \lambda_j = 1:$$

Min $\qquad \rho = \dfrac{1 - \frac{1}{m}\sum_{i=1}^{m} \frac{s_i^-}{x_{io}}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{s_i^+}{y_{ro}}}$

Subject to

$$\sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}. \qquad i = 1.2. \dots. m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}. \qquad r = 1.2. \dots. s$$

$$\sum_{j=1}^{n} \lambda_j = 1.$$

$$\lambda_j. s_i^-. s_r^+ \geq 0. \qquad \forall_j. i \ and \ r.$$

where o represents the DMU to be assessment, and s_i^- ($i = 1,2,...,m$) are input excess variable and s_r^+ ($r = 1,2,...,s$) are output shortfall variable. It is worth mentioning that

$0 \leq \rho \leq 1$ and a DMU (x_o, y_o) is a SBM-efficient if and only if $\rho^* = 1$. We know that a BCC model can be formulated as follows:

$$min \ \theta$$
$$\text{s.t.}$$
$$\theta x_0 = X\mu + t^- \tag{1.1}$$

$$y_0 = Y\mu - t^+ \tag{1.2}$$
$$e\mu = 1$$
$$\mu \geq 0, t^- \geq 0, t^+ \geq 0$$

Let the optimal solution derived from the above BCC model be $(\theta, \mu^*, t^{-*}, and \ t^{+*})$.

From equation (1.1) we can derive

$$x_0 = X_{\mu^*} + t^{-*} + (1 - \theta^*)x_0 \tag{1.3}$$

$$y_0 = Y_{\mu^*} - t^{-*} \tag{1.4}$$

and define

$$\lambda = \mu^* \tag{1.5}$$

$$s^- = t^{-*} + (1 - \theta)x_0 \tag{1.6}$$

$$s^+ = t^{+*} \tag{1.7}$$

then (λ, s^-, s^+) is feasible for a SBM model. If we insert equations (1.6) and (1.7) into the main objective function, its optimum value is expressed as

$$\rho = \frac{1 - \frac{1}{m}\left[\sum_{i=1}^{m} \frac{t_i^{-*}}{x_{i0}} + m(1 - \theta^*)\right]}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{t_r^{+*}}{y_{r0}}} = \frac{\theta^* - \frac{1}{m}\sum_{i=1}^{m} \frac{t_i^{-*}}{x_{i0}}}{1 + \frac{1}{s}\sum_{r=1}^{s} \frac{t_r^{+*}}{y_{r0}}}$$

It is worth mentioning that the optimum value of ρ holds if $\theta^* > \frac{1}{m}\sum_{i=1}^{m} \frac{t_i^{-*}}{x_{i0}}$. By comparing the optimum values of the two models, i.e., the SBM_{ρ^*} and BCC_{ρ^*}, models we can conclude that the optimal SBM_{ρ^*} is not greater than the optimal BCC_{ρ^*} which implies that $\rho_{SBM}^* \leq \rho_{BCC}^*$.

In the following, some SBM models with ratio as output are presented. First, the standard input-oriented SBM model is as follows:

Min $\qquad \rho = 1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}}$

Subject to

$$x_{io} = \sum_{j=1}^{n}\lambda_j x_{ij} + s_i^-, \qquad i = 1.2.\,\dots\,m$$
$$y_{ro} \le \sum_{j=1}^{n}\lambda_j y_{rj}, \qquad r = 1.2.\,\dots\,s$$
$$\sum_{j=1}^{n}\lambda_j = 1,$$
$$\lambda_j \ge 0 \qquad j = 1.2.\,\dots\,n$$
$$s_i^- \ge 0 \qquad i = 1.2.\,\dots\,m$$

In the above model, y_{kj} for unit j is calculated from the numerator and denominator of $\bar{y}_{kj}$ and $\underline{y}_{kj}$ respectively, i.e. $y_{kj} = \frac{\bar{y}_{kj}}{\underline{y}_{kj}}$ $(j = 1.2.\,\dots\,n)$. Thus, in this input-oriented SBM model's solution, the numerator and denominator of the output-ratio variables are presented as output and input separately.

Second, the standard input-oriented SBM model that evaluates the efficiency score of DMUs against n DMUs is presented. It is worthy to mention that each one of DMUs contains $m + 1$ input and s output.

Min $\qquad \rho = 1 - \frac{1}{m+1}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}} + \frac{s_{k2}}{\underline{y}_{kj}}$

Subject to

$$x_{io} = \sum_{j=1}^{n}\lambda_j x_{ij} + s_i^-, \qquad i = 1.2.\,\dots\,m$$
$$y_{ro} \le \sum_{j=1}^{n}\lambda_j y_{rj}, \qquad r \ne k$$
$$\underline{y}_{ko} = \sum_{j=1}^{n}\lambda_j \underline{y}_{kj} + s_{k2}^-,$$
$$\bar{y}_{ko} \le \sum_{j=1}^{n}\lambda_j \bar{y}_{kj}$$
$$\sum_{j=1}^{n}\lambda_j = 1,$$
$$\lambda_j \ge 0 \qquad j = 1.2.\,\dots\,n$$
$$s_i^- \cdot s_{k2}^- \ge 0 \qquad i = 1.2.\,\dots\,m$$

In the above model, s_{k2} shows an excess input variable when denominator of the output variable is used as an input variable in model. Thus, in this input-oriented SBM model's solution, the correct convexity for the ratio variable is taken into account. It is worth mentioning that in the standard SBM the convex combination of DMUs for the kth-output is defined as:

$$\sum_{j=1}^{n} y_{kj}\lambda_j = \sum_{j=1}^{n} \frac{\bar{y}_{kj}}{\underline{y}_{kj}}\lambda_j$$

However, in the correct convex combination, the kth-output is included into the model as follows:

$$\frac{\sum_{j=1}^{n}\bar{y}_{kj}\lambda_j}{\sum_{j=1}^{n}\underline{y}_{kj}\lambda_j}$$

which implies that the convexity assumption (when assessing unit o) should be taken into the model as follows:

$$\frac{\sum_{j=1}^{n}\bar{y}_{kj}\lambda_j}{\sum_{j=1}^{n}\underline{y}_{kj}\lambda_j} \ge \frac{\bar{y}_{ko}}{\underline{y}_{ko}} = y_{ko}$$

Therefore, the input-oriented SBM model with output ratio can be presented as follows:

Min $\quad \rho = 1 - \frac{1}{m}\sum_{i=1}^{m}\frac{s_i^-}{x_{io}}$

Subject to

$$x_{io} = \sum_{j=1}^{n}\lambda_j x_{ij} + s_i^- \qquad\qquad i = 1.2.\dots.m$$
$$y_{ro} \leq \sum_{j=1}^{n}\lambda_j y_{rj} \qquad\qquad r \neq k$$
$$\sum_{j=1}^{n}\bar{y}_{kj}\lambda_j - y_{ko}\sum_{j=1}^{n}\underline{y}_{kj}\lambda_j \geq 0. \qquad r = k.$$
$$\sum_{j=1}^{n}\lambda_j = 1$$
$$\lambda_j \geq 0 \qquad\qquad\qquad j = 1.2.\dots.n$$
$$s_i^- \geq 0 \qquad\qquad\qquad i = 1.2.\dots.m$$

4 Empirical Analysis

In this section, we presented a numerical example which encompasses a case with mixed output, absolute, and output-ratio that shows the efficiency scores obtained from a modified SBM model is not greater than the ones we found in [11] from conducting the modified BCC models.

Let consider a scenario in which 10 universities used a single input such as total expenditure in million dollars (x) to produce a couple of outputs including y_1 = percentage degree awarded (i.e., in an output-ratio form), and y_2 = amounts of research income in million dollars (Table 1). Moreover, suppose that the percentage degree awarded variable, y_1, is obtained from the following equation:

$$Percentage\ degree\ awarded\ for\ U_j = \frac{Number\ of\ degree\ awarded\ for\ U_j}{total\ number\ of\ students\ for\ U_j}$$

Table 1. The university data

DMU university	x_1 Cost	$\underline{y}_1$ Number of student	$\bar{y}_1$ Number of degree awarded	y_1 %degree awarded	y_2 Amounts of research income(million)
U_1	165	6500	1300	0.20	30
U_2	200	8000	2000	0.25	40
U_3	80	2000	300	0.15	12
U_4	120	4000	720	0.18	30
U_5	150	6000	600	0.10	30
U_6	210	10000	2500	0.25	45
U_7	165	2500	500	0.20	30
U_8	105	3000	540	0.18	40
U_9	300	13000	5200	0.40	60
$U10$	90	2000	320	0.16	12

Source: Sample data

Using the methodologies discussed in the previous section, we calculated the efficiency score obtained from both modified BCC and SBM models, which are shown in Tables 2 and 3, respectively.

By comparing the results obtained from the two models and shown in Tables 2 and 3, we can conclude that the efficiency scores obtained from the SBM models are not greater than the ones obtained from the BCC models, i.e., $\rho_{SBM}^* \leq \rho_{BCC}^*$.

Table 2. Efficiency score comparison corresponding with BCC models (1)-(3)

University	Efficiency score in model (1) Input: x_1 Outputs: y_1, y_2	Efficiency score in model (2) Inputs: x_1, $\underline{y_1}$ Outputs: $\overline{y_1}$, y_2	Efficiency score in model (3) Input: x_1 Outputs: y_1, y_2
U_1	74.38	78.45	61.26
U_2	83.52	81.31	61.08
U_3	100	100	100
U_4	87.50	88.8	81.18
U_5	64.05	67.88	64.05
U_6	79.55	88.15	73.21
U_7	74.38	100	61.26
U_8	100	100	100
U_9	100	100	100
$U10$	98.15	100	90.45

Source: Sample data

Table 3. Efficiency score comparison with SBM models (1)-(3)

University	Efficiency score in model (1) Input: x_1 Outputs: y_1, y_2	Efficiency score in model (2) Inputs: x_1, $\underline{y_1}$ Outputs: $\overline{y_1}$, y_2	Efficiency score in model (3) Input: x_1 Outputs: y_1, y_2
U_1	74.38	73.01	61.26
U_2	83.52	78.28	61.08
U_3	100	100	100
U_4	87.50	84.31	81.18
U_5	64.05	58.48	64.05
U_6	79.55	79.80	73.21
U_7	74.38	100	61.26
U_8	100	100	100
U_9	100	100	100
$U10$	98.15	100	90.45

Source: Sample data

5 Conclusion

The literature has documented substantial studies in the field of estimating technical efficiency of decision making units using both parametric and non-parametric and deterministic and stochastic frontier analysis. Amongst all the non-parametric methods of measuring technical efficiency of production units or decision making the data envelopment analysis (DEA) and free disposal hull (FDH) have recently been much paid attention by practitioners. DEA is a linear programming methodology used to construct a piece-wise convex surface (or frontier) which "envelops" the data [14]. FDH is similar to DEA, but the convexity assumption is relaxed [23]. With both of these deterministic frontier methods, the distance from each observation to the computed frontier is the measure of inefficiency [8]. In this study, we proposed slack-based measure model on the basis of production possibility set of the Banker, Charnes, and Cooper model to measure the efficiency of decision making units (DMUs) when an input/output ratio variable is included in the model [13]. We also demonstrated the characteristics of the model theoretically and presented empirical application of the model by providing a numerical example. This study shows a significant contribution to the literature by modifying conventional solutions for DEA models used for calculating efficiency of decision making units through slack-based-measure models when input-output ratios are included in the model as

independent variables. From economists' perspectives, studies like this when the DEA method is used to measure technical efficiency of decisions making units are not without drawbacks. The major shortcoming of the DEA method is that it is not a fully statistical method and does not account for ransom noise in the data. Instead, other stochastic non-parametric frontier analysis methods such as generalized additive models (GAMs) proposed by Hastie and Tibshirani [24] and generalized additive mixed models (GAMMs) proposed by Lin and Zhang [25] are recommended [4]. Some direction for future research includes the study of the super-efficiency and sensitivity analysis of decision making units with non-ratio data by using slacks-based measure of super efficiency.

Competing Interests

Authors have declared that no competing interests exist.

References

[1]   Cook WD, Tone K, Zhu J. Data envelopment analysis: Prior to choosing a model. The International Journal of Management Science (OMEGA). 2014 ;44:1-4.

[2]   Cooper WW, Seiford LM, Tone K. Introduction to data envelopment analysis and its uses: with DEA-solver software and references, 1st Edition. Springer: New York, U.S.A; 2006.

[3]   Sherman HD, Zhu J. Analyzing performance in service organizations. Sloan Management Review. 2013;54(4):37-42.

[4]   Haghiri M, Law SM, Simchi A. Testing for input separability in the Canadian CATV industry: An application of generalized additive mixed models. British Journal of Economics, Management and Trade. 2016;15(2):1-9.

[5]   Haghiri M, Law SM, Nolan JF, Simchi A. Comparing non-parametric to parametric interaction terms in generalized additive models: Production technology in the Canadian cable television industry. International Journal of Data Analysis Techniques and Strategies. 2013;5(3):229-251.

[6]   Haghiri M, Law SM, Nolan JF. Nonparametric estimates of technology using generalized additive models: The case of the Canadian cable television industry. International Journal of Economics and Finance. 2012;4(1):36-47.

[7]   Haghiri M, Simchi A. A nonparametric extension of generalized quadratic Box-Cox models in measuring technical efficiency. Applied Economics. 2010;42(1):1-8.

[8]   Haghiri M, Nolan JF, Tran KC. Assessing the impact of economic liberalization across countries: A comparison of dairy industry efficiency in Canada and the United States. Applied Economics. 2004;36(11):1233-1243.

[9]   Law SM, Haghiri M, Nolan JF. Evaluating the relative efficiency of cancer treatments in Nova Scotia using the patient as the unit of analysis. Canadian Journal of Regional Science. 2011;33(3):163-176.

[10]   Charnes A, Cooper WW, Lewin AY, Seiford LM. Data envelopment analysis: Theory, methodology, and application. Boston: Kluwer Academic Publishers; 1994.

[11]   Emrouznejad A, Parker B, Tavares G. Evaluation of research in efficiency and productivity: A survey and analysis of the first 30 years of scholarly literature in DEA. Socio-Economic Planning Sciences. 2008;42(3):151-157.

[12]   Emrouznejad A, Amin GR. DEA model for ratio data: convexity consideration. Applied Mathematical Modeling. 2009;33(1):486-498.

[13] Banker RD, Charnes A, Cooper WW. Some models for estimating technical and scale inefficiencies in data envelopment analysis. Management Science. 1984;30(9):1078-1092.

[14] Charnes A, Cooper WW, Rhodes E. A data envelopment analysis approach to evaluation of the program follow through experiments in U.S. public school education. Management Science Research Report No. 432, Carnegie-Mellon University, School of Urban and Public Affairs, Pittsburgh, PA; 1978.

[15] Hollingsworth B, Smith P. The use of ratio in data envelopment analysis. Applied Economics Letters. 2003;10(11):733-735.

[16] Bian Y, Hu M, Wang Y, Xu H. Energy efficiency analysis of the economic system in China during 1986-2012: A parallel slacks-based measure approach. Renewable and Sustainable Energy Reviews. 2016;55:990-998.

[17] Bian Y, Liang N, Xu H. Efficiency evaluation of Chinese regional industrial systems with undesirable factors using a two-stage slacks-based measure approach. Journal of Cleaner Production. 2015;87: 348-356.

[18] Lee T, Yeo G, Thai VV. Environmental efficiency analysis of port cities: Slacks-based measure data envelopment analysis approach. Journal of Transport Policy. 2014;33:82-88.

[19] Akther S, Fukuyama H, Weber WL. Estimating two-stage network slacks-based inefficiency: An application to Bangladesh banking. Omega: The International Journal of Management Science. 2013;41:88-96.

[20] Fukuyama H, Weber WL. A slacks-based inefficiency measure for a two-stage system with bad outputs. Omega: The International Journal of Management Science. 2010;38(5):239-410.

[21] Chen L, Lai F, Wang Y-M, Huang Y, Wu F-M. A two-stage network data envelopment approach for measuring and decomposing environmental efficiency. Journal of Computers and Industrial Engineering. 2018;119:388-403.

[22] Tone K. A slacks based measure of efficiency in data envelopment analysis. European Journal of Operational Research. 2001;30(3):498-509.

[23] Deprins D, Simar L, Tulkens H. Measuring labor-efficiency in post offices. In M. Marchand, P. Pestieau, and H. Tulkens (Eds.), The Performance of Public Enterprises: Concepts and Measurement, Amsterdam: North Holland. 1984;243-267.

[24] Hastie TJ, Tibshirani RJ. Generalized additive models. 1st Edition, London: Chapman and Hall; 1990.

[25] Lin X, Zhang D. Inferences in generalized additive mixed models by using smoothing splines. Journal of Royal Statistical Society Series B. 1999;61(2):381-400.