

Common Fixed Point Theorems in Intuitionistic Fuzzy Metric Spaces

Saurabh Manro¹, Sanjay Kumar², Shivdeep Singh³

¹School of Mathematics and Computer Applications, Thapar University, Patiala, India

²Deenbandhu Chhotu Ram, University of Science and Technology, Sonapat, India

³Vinayaka Missions University, Salem Tamilnadu, India

E-mail: sauravmanro@yahoo.com

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Abstract

In this paper, we introduce the concept of ϵ -chainable intuitionistic fuzzy metric space akin to the notion of ϵ -chainable fuzzy metric space introduced by Cho, and Jung [1] and prove a common fixed point theorem for weakly compatible mappings in this newly defined space.

Keywords: ϵ -Chainable Intuitionistic Fuzzy Metric Space, Weakly Compatible Maps.

1. Introduction

The human reasoning involves the use of variable whose values are fuzzy sets. Description of system behavior in the language of fuzzy rules lowers the need for precision in data gathering and data manipulation, and in effect may be viewed as a form of data compression. But there are situations when description by a (fuzzy) linguistic variable given in terms of a membership function only, seems too rough. The use of linguistic variables represents a physical significant paradigm shift in system analysis.

Atanassov [2] introduced the notion of intuitionistic fuzzy sets by generalizing the notion of fuzzy set by treating membership as a fuzzy logical value rather than a single truth value. For an intuitionistic set the logical value has to be consistent (in the sense $\gamma_A(x) + \mu_A(x) \geq 1$). $\gamma_A(x)$ and $\mu_A(x)$ denotes degree of membership and degree of non-membership, respectively. All results which hold of fuzzy sets can be transformed Intuitionistic fuzzy sets but converse need not be true. Intuitionistic fuzzy set can be viewed in the context as a proper tool for representing hesitancy concerning both membership and non-membership of an element to a set. To be more precise, a basic assumption of fuzzy set theory that if we specify the degree of membership of an element in a fuzzy set as a real number from $[0, 1]$, say 'a', then the degree of its non-membership is automatically determined as '(1 - a)', need not hold for intuitionistic fuzzy sets. In intuitionistic fuzzy set theory it is assumed that

non-membership should not be more than $(1 - a)$. For instant, lack of knowledge (hesitancy concerning both membership and non-membership of an element to a set) and the temperature of a patient changes and other symptoms are not quite clear. The area of intuitionistic fuzzy image processing is just beginning to develop; there are hardly few methods in the literature. Intuitionistic fuzzy set theory has been used to extract information by reflecting and modeling the hesitancy present in real-life situations. The application of Intuitionistic fuzzy sets instead of fuzzy sets means the introduction of another degree of freedom into a set description. By employing intuitionistic fuzzy sets in databases we can express a hesitancy concerning examined objects.

Coker [3] introduced the concept of intuitionistic fuzzy topological spaces. Alaca *et al.* [4] proved the well-known fixed point theorems of Banach [5] in the setting of intuitionistic fuzzy metric spaces. Later on, Turkoglu *et al.* [6] proved Jungck's [7] common fixed point theorem in the setting of intuitionistic fuzzy metric space. Turkoglu *et al.* [6] further formulated the notions of weakly commuting and R-weakly commuting mappings in intuitionistic fuzzy metric spaces and proved the intuitionistic fuzzy version of Pant's theorem [8]. Gregori *et al.* [9], Saadati and Park [10] studied the concept of intuitionistic fuzzy metric space and its applications. No wonder that intuitionistic fuzzy fixed point theory has become an area of interest for specialists in fixed point theory as intuitionistic fuzzy mathematics has covered new possibilities for fixed point theorists. Recently,

many authors have also studied the fixed point theory in fuzzy and intuitionistic fuzzy metric spaces (see [1], [11-15]).

In 2006, Cho, Jung [1] introduced the notion of ϵ -chainable fuzzy metric space and prove common fixed point theorems for four weakly compatible mappings. In a similar mode, we introduce the concept of ϵ -chainable intuitionistic fuzzy metric space and prove common fixed point theorems for four weakly compatible mappings of ϵ -chainable intuitionistic fuzzy metric space.

2. Preliminaries

We begin by briefly recalling some definitions and notions from fixed point theory literature that we will use in the sequel. The concepts of triangular norms (t-norm) and triangular conorms (t-conorm) were originally introduced by Schweizer and Sklar [16].

Definition 2.1 [16] A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t-norm if $*$ is satisfying the following conditions:

- 1) $*$ is commutative and associative;
- 2) $*$ is continuous
- 3) $a * 1 = a$ for all $a \in [0, 1]$;
- 4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all

$a, b, c, d \in [0, 1]$.

Definition 2.2 [16] A binary operation \diamond : $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t-conorm if \diamond is satisfying the following conditions:

- 1) \diamond is commutative and associative;
- 2) \diamond is continuous;
- 3) $a \diamond 0 = a$ for all $a \in [0, 1]$;
- 4) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for all

$a, b, c, d \in [0, 1]$.

Definition 2.3 [4] A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, \diamond is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions:

- 1) $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X$ and $t > 0$;
- 2) $M(x, y, 0) = 0$ for all $x, y \in X$;
- 3) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
- 4) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;
- 5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;
- 6) for all $x, y \in X$, $M(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$ is left continuous;
- 7) $\lim_{t \rightarrow \infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$;
- 8) $N(x, y, 0) = 1$ for all $x, y \in X$;
- 9) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

10) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;

11) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;

12) for all $x, y \in X$, $N(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$ is right continuous;

13) $\lim_{t \rightarrow \infty} N(x, y, t) = 0$ for all $x, y \in X$.

(M, N) is called an intuitionistic fuzzy metric on X . The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t , respectively.

Remark 2.1 [17] An intuitionistic fuzzy metric spaces with continuous t-norm $*$ and continuous t-conorm \diamond defined by $a * a \geq a$ and $(1 - a) \diamond (1 - a) \leq (1 - a)$ for all $a \in [0, 1]$. Then for all $x, y \in X$, $M(x, y, *)$ is non-decreasing and $N(x, y, \diamond)$ is non-increasing.

Alaca, Turkoglu and Yildiz [4] introduced the following notions:

Definition 2.4 [4] Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. Then

1) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$, $\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 1$, $\lim_{n \rightarrow \infty} N(x_{n+p}, x_n, t) = 0$.

2) a sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if, for all $t > 0$, $\lim_{n \rightarrow \infty} M(x_n, x, t) = 1$, $\lim_{n \rightarrow \infty} N(x_n, x, t) = 0$.

Since $*$ and \diamond are continuous, the limit is uniquely determined from 5) and 11) of Definition 2.3, respectively. An intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Turkoglu, Alaca and Yildiz [15] introduced the notions of compatible mappings in intuitionistic fuzzy metric space, akin to the concept of compatible mappings introduced by Jungck [18] in metric spaces as follows:

Definition 2.5 [15] A pair of self-mappings (f, g) of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be compatible if

$\lim_{n \rightarrow \infty} M(fg x_n, g f x_n, t) = 1$ and $\lim_{n \rightarrow \infty} N(fg x_n, g f x_n, t) = 0$ for every $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} f x_n = \lim_{n \rightarrow \infty} g x_n = z$ for some $z \in X$.

Definition 2.6 [15] A pair of self-mappings (f, g) of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be non-compatible if $\lim_{n \rightarrow \infty} M(fg x_n, g f x_n, t) \neq 1$ or non-existent and $\lim_{n \rightarrow \infty} N(fg x_n, g f x_n, t) \neq 0$ or non-existent for every $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} f x_n = \lim_{n \rightarrow \infty} g x_n = z$ for some $z \in X$.

In 1998, Jungck and Rhoades [19] introduced the concept of weakly compatible maps as follows:

Definition 2.7 [19] Two self maps f and g are said to be weakly compatible if they commute at coincidence points.

Lemma 2.1 Let $(X, M, N, *, \diamond)$ be intuitionistic fuzzy

metric space and for all x, y in $X, t > 0$ and if for a number k in $(0, 1)$,

$M(x, y, kt) \geq M(x, y, t)$ and $N(x, y, kt) \leq N(x, y, t)$
Then $x = y$.

Now, we define ϵ -chainable intuitionistic fuzzy metric space as follows:

Definition 2.8 Let $(X, M, N, *, \diamond)$ be intuitionistic fuzzy metric space. A finite sequence

$x = x_0, x_1, x_2, \dots, x_n = y$ is called ϵ -chain from x to y if there exists a positive number $\epsilon > 0$ such that

$M(x_i, x_{i-1}, t) > 1 - \epsilon$ and $N(x_i, x_{i-1}, t) < 1 - \epsilon$ for all $t > 0$ and $i = 1, 2, \dots, n$.

An intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is called ϵ -chainable if for any $x, y \in X$, there exists an ϵ -chain from x to y .

3. Main Results

Theorem 3.1 Let A, B, S and T be self maps of a complete ϵ -chainable intuitionistic fuzzy metric spaces $(X, M, N, *, \diamond)$ with continuous t -norm $*$ and continuous t -conorm \diamond defined by

$a * a \geq a$ and $(1 - a) \diamond (1 - a) \leq (1 - a)$ for all $a \in [0, 1]$ satisfying the following condition:

- 1) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$,
- 2) A and S are continuous,
- 3) the pairs (A, S) and (B, T) are weakly compatible,
- 4) there exist $q \in (0, 1)$ such that

$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)$ and

$N(Ax, By, qt) \leq N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(By, Ty, t) \diamond N(Ax, Ty, t)$, for every x, y in X and $t > 0$.

Then A, B, S and T have a unique common fixed point in X .

Proof. As $A(X) \subseteq T(X)$, for any $x_0 \in X$, there exists a point $x_1 \in X$ such that $Ax_0 = Tx_1$.

Since $B(X) \subseteq S(X)$, for this point x_1 , we can choose a point $x_2 \in X$ such that $Bx_1 = Sx_2$. Inductively, we can find a sequence $\{y_n\}$ in X as follows:

$y_{2n-1} = Tx_{2n-1} = Ax_{2n-2}$ and $y_{2n} = Sx_{2n} = Bx_{2n-1}$ for $n = 1, 2, \dots$

By Theorem of Alaca *et al.* [4], we can conclude that $\{y_n\}$ is Cauchy sequence in X .

Since X is complete, therefore sequence $\{y_n\}$ in X converges to z for some z in X and so the sequences $\{Tx_{2n-1}\}, \{Ax_{2n-2}\}, \{Sx_{2n}\}$ and $\{Bx_{2n-1}\}$ also converges to z .

Since X is ϵ -chainable, there exists ϵ -chain from x_n to x_{n+1} , that is, there exists a finite sequence

$x_n = y_1, y_2, \dots, y_l = x_{n+1}$ such that

$M(y_i, y_{i-1}, t) > 1 - \epsilon$ and $N(y_i, y_{i-1}, t) < 1 - \epsilon$ for all $t > 0$ and $i = 1, 2, \dots, l$. Thus we have

$M(x_n, x_{n+1}, t) \geq M(y_1, y_2, t/l) * M(y_2, y_3, t/l) * \dots * M(y_{l-1}, y_l, t/l) > (1 - \epsilon) * (1 - \epsilon) * \dots * (1 - \epsilon) \geq (1 - \epsilon)$ and

$N(x_n, x_{n+1}, t) \leq N(y_1, y_2, t/l) \diamond N(y_2, y_3, t/l) \diamond \dots \diamond N(y_{l-1}, y_l, t/l) < (1 - \epsilon) \diamond (1 - \epsilon) \diamond \dots \diamond (1 - \epsilon) \leq (1 - \epsilon)$

For $m > n$,

$M(x_n, x_m, t) \geq M(x_n, x_{n+1}, t/m-n) * M(x_{n+1}, x_{n+2}, t/m-n) * \dots * M(x_{m-1}, x_m, t/m-n) > (1 - \epsilon) * (1 - \epsilon) * \dots * (1 - \epsilon) \geq (1 - \epsilon)$ and

$N(x_n, x_m, t) \leq N(x_n, x_{n+1}, t/m-n) \diamond N(x_{n+1}, x_{n+2}, t/m-n) \diamond \dots \diamond N(x_{m-1}, x_m, t/m-n) < (1 - \epsilon) \diamond (1 - \epsilon) \diamond \dots \diamond (1 - \epsilon) \leq (1 - \epsilon)$

Therefore, $\{x_n\}$ is a Cauchy sequence in X and hence there exists x in X such that $x_n \rightarrow x$. From 2), $Ax_{2n-2} \rightarrow Ax, Sx_{2n} \rightarrow Sx$ as limit $n \rightarrow \infty$. By uniqueness of limits, we have $Ax = z = Sx$. Since pair (A, S) is weakly compatible, therefore, $ASx = SAx$ and so $Az = Sz$.

From 2) of Theorem 3.1, we have $ASx_{2n} \rightarrow ASx$ and therefore, $ASx_{2n} \rightarrow Sz$. Also, from continuity of S , we have, $SSx_{2n} \rightarrow Sz$.

From 4), we get

$M(ASx_{2n}, Bx_{2n-1}, qt) \geq M(SSx_{2n}, Tx_{2n-1}, t) * M(ASx_{2n}, SSx_{2n}, t) * M(Bx_{2n-1}, Tx_{2n-1}, t) *$

$M(ASx_{2n}, Tx_{2n-1}, t)$ and

$N(ASx_{2n}, Bx_{2n-1}, qt) \leq N(SSx_{2n}, Tx_{2n-1}, t) \diamond$

$N(ASx_{2n}, SSx_{2n}, t) \diamond N(Bx_{2n-1}, Tx_{2n-1}, t) \diamond$

$N(ASx_{2n}, Tx_{2n-1}, t)$

Proceeding limit as $n \rightarrow \infty$, we have

$M(Sz, z, qt) \geq M(Sz, z, t) * M(Sz, Sz, t) * M(z, z, t) * M(Sz, z, t)$ and

$N(Sz, z, qt) \leq N(Sz, z, t) \diamond N(Sz, Sz, t) \diamond N(z, z, t) \diamond N(Sz, z, t)$.

From Lemma 2.1, we get $Sz = z$, and hence

$Az = Sz = z$.

Since $A(X) \subseteq T(X)$, there exists v in X such that

$Tv = Az = z$.

From 4), we have

$M(Ax_{2n}, Bv, qt) \geq M(Sx_{2n}, Tv, t) * M(Ax_{2n}, Sx_{2n}, t) * M(Bv, Tv, t) * M(Ax_{2n}, Tv, t)$ and

$N(Ax_{2n}, Bv, qt) \leq N(Sx_{2n}, Tv, t) \diamond N(Ax_{2n}, Sx_{2n}, t) \diamond N(Bv, Tv, t) \diamond N(Ax_{2n}, Tv, t)$.

Letting $n \rightarrow \infty$, we have

$M(z, Bv, qt) \geq M(z, Tv, t) * M(z, z, t) * M(Bv, Tv, t) * M(z, Tv, t)$

$= M(z, z, t) * M(z, z, t) * M(Bv, z, t) * M(z, z, t) \geq M(Bv, z, t)$ and

$N(z, Bv, qt) \leq N(z, Tv, t) \diamond N(z, z, t) \diamond N(Bv, Tv, t) \diamond N(z, Tv, t)$

$= N(z, z, t) \diamond N(z, z, t) \diamond N(Bv, z, t) \diamond N(z, z, t) \geq N(Bv, z, t)$.

By Lemma 2.1, we have $Bv = z$ and therefore, we have $Tv = Bv = z$.

Since (B, T) is weakly compatible, therefore, $TBv = BTv$ and hence $Tz = Bz$.

From 4),

$$M(Ax_{2n}, Bz, qt) \geq M(Sx_{2n}, Tz, t) * M(Ax_{2n}, Sx_{2n}, t) * M(Bz, Tz, t) * M(Ax_{2n}, Tz, t) \text{ and}$$

$$N(Ax_{2n}, Bz, qt) \leq N(Sx_{2n}, Tz, t) \diamond N(Ax_{2n}, Sx_{2n}, t) \diamond N(Bz, Tz, t) \diamond N(Ax_{2n}, Tz, t),$$

Letting $n \rightarrow \infty$, we have

$$M(z, Bz, qt) \geq M(z, Tz, t) * M(z, z, t) * M(Bz, Tz, t) * M(z, Tz, t)$$

$$= M(z, Bz, t) * M(z, z, t) * M(Bz, Bz, t) * M(z, Bz, t) \geq M(z, Bz, t)$$

and

$$N(z, Bz, qt) \leq N(z, Tz, t) \diamond N(z, z, t) \diamond N(Bz, Tz, t) \diamond N(z, Tz, t)$$

$$= N(z, Bz, t) \diamond N(z, z, t) \diamond N(Bz, Bz, t) \diamond N(z, Bz, t) \geq N(z, Bz, t),$$

which implies that $Bz = z$.

Therefore, $Az = Sz = Bz = Tz = z$. Hence A, B, S and T have common fixed point z in X .

For uniqueness, let w be another common fixed point of A, B, S and T .

$$\text{Then } M(z, w, qt) = M(Az, Bw, qt) \geq M(Sz, Tw, t) * M(Az, Sz, t) * M(Bw, Tw, t) * M(Az, Tw, t) \geq M(z, w, t) \text{ and}$$

$$N(z, w, qt) = N(Az, Bw, qt) \geq N(Sz, Tw, t) \diamond$$

$$N(Az, Sz, t) \diamond N(Bw, Tw, t) \diamond N(Az, Tw, t) \leq N(z, w, t).$$

From Lemma 2.1, we conclude that $z = w$. Hence A, B, S and T have unique common fixed point z in X .

Corollary 3.1. Let A, B, S and T be self maps of a complete ϵ -chainable intuitionistic fuzzy metric spaces $(X, M, N, *, \diamond)$ with continuous t -norm $*$ and continuous t -conorm \diamond defined by $a * a \geq a$ and $(1 - a) \diamond (1 - a) \leq (1 - a)$ for all $a \in [0, 1]$ satisfying 1), 2), 3) of Theorem 3.1 the following:

there exists $q \in (0, 1)$ such that

$$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Sx, By, 2t) * M(By, Ty, t) * M(Ax, Ty, t)$$

$$\text{and } N(Ax, By, qt) \leq N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(Sy, By, 2t) \diamond N(By, Ty, t) \diamond N(Ax, Ty, t) \text{ for every } x, y \text{ in } X \text{ and } t > 0.$$

Then A, B, S and T have a unique common fixed point in X .

Corollary 3.2. Let A, B, S and T be self maps of a complete ϵ -chainable intuitionistic fuzzy metric spaces $(X, M, N, *, \diamond)$ with continuous t -norm $*$ and continuous t -conorm \diamond defined by $a * a \geq a$ and $(1 - a) \diamond (1 - a) \leq (1 - a)$ for all $a \in [0, 1]$ satisfying 1), 2), 3) of Theorem 3.1 the following:

there exists $q \in (0, 1)$ such that

$$M(Ax, By, qt) \geq M(Sx, Ty, t) \text{ and } N(Ax, By, qt) \leq N(Sx, Ty, t)$$

for every x, y in X and $t > 0$. Then A, B, S and T have a unique common fixed point in X .

Corollary 3.3. Let $(X, M, N, *, \diamond)$ be complete ϵ -

chainable intuitionistic fuzzy metric space and let A and B be self mappings of X satisfying the following condition: there exists $q \in (0, 1)$ such that

$$M(Ax, By, qt) \geq M(x, y, t) \text{ and } N(Ax, By, qt) \leq N(x, y, t)$$

for every x, y in X and $t > 0$. Then A and B have a unique common fixed point in X provided pair (A, B) is weakly compatible map.

Proof. Take S and T be identity mapping on set X in Corollary 3.2.

Corollary 3.4. Let $(X, M, N, *, \diamond)$ be complete ϵ -chainable intuitionistic fuzzy metric space and let A be self mappings of X satisfying the following condition: there exists

$q \in (0, 1)$ such that

$$M(Ax, Ay, qt) \geq M(x, y, t) \text{ and } N(Ax, Ay, qt) \leq N(x, y, t)$$

for every x, y in X and $t > 0$. Then A has a unique common fixed point in X .

Proof. Take $A = B$ in Corollary 3.3, we get desired result.

5. References

- [1] S. H. Cho and J. H. Jung, "On Common Fixed Point Theorems in Fuzzy Metric Spaces," *International Mathematical Forum*, Vol. 1, No. 29, 2006, pp. 1441-1451.
- [2] K. Atanassov, "Intuitionistic Fuzzy Sets," *Fuzzy Sets and System*, Vol. 20, No. 1, 1986, pp. 87-96.
- [3] D. Coker, "An Introduction to Intuitionistic Fuzzy Topological Spaces," *Fuzzy Sets and System*, Vol. 88, No. 1, 1997, pp. 81-89.
- [4] C. Alaca, D. Turkoglu and C. Yildiz, "Fixed Points in Intuitionistic Fuzzy Metric Spaces," *Chaos, Solitons and Fractals*, Vol. 29, No. 5, 2006, pp. 1073-1078.
- [5] S. Banach, "Theorie Les Operations Lineaires, Manograie Matematyeczne Warsaw Poland," In French, Z Subwencji Funduszu Kultury Narodowej, New York, 1932.
- [6] D. Turkoglu, C. Alaca, Y. J. Cho and C. Yildiz, "Common Fixed Point Theorems in Intuitionistic Fuzzy Metric spaces," *Journal of Applied Mathematics and Computing*, Vol. 22, No. 1-2, 2006, pp. 411-424.
- [7] G. Jungck, "Commuting Mappings and Fixed Points," *American Mathematical Monthly*, Vol. 83, No. 4, 1976, pp. 261-263.
- [8] R. P. Pant, "Common Fixed Points of Noncommuting Mappings," *Journal of Mathematical Analysis and Applications*, Vol. 188, No. 2, 1994, pp. 436-440.
- [9] V. Gregori, S. Romaguera and P. Veeramani, "A Note on Intuitionistic Fuzzy Metric Spaces," *Chaos, Solitons and Fractals*, Vol. 28, No. 4, 2006, pp. 902-905.
- [10] R. Saadati and J. H. Park, "On the Intuitionistic Fuzzy Topological Spaces," *Chaos, Solitons and Fractals*, Vol. 27, No. 2, 2006, pp. 331-344.
- [11] M. Grabiec, "Fixed Points in Fuzzy Metric Spaces," *Fuzzy Sets and Systems*, Vol. 27, No. 3, 1988, pp. 385-389.

- [12] M. Imdad and Javid Ali, "Some Common Fixed Point Theorems in Fuzzy Metric Spaces," *Mathematical Communication*, Vol. 11, No. 12, 2006, pp. 153-163.
- [13] J. H. Park, "Intuitionistic Fuzzy Metric Spaces," *Chaos, Solitons & Fractals*, Vol. 22, No. 52004, pp. 1039-1046.
- [14] J. S. Park, Y. C. Kwun and J. H. Park, "A Fixed Point Theorem in the Intuitionistic Fuzzy Metric Spaces," *Far East Journal of Mathematical Sciences*, Vol. 16, No. 2, 2005, pp. 137-149.
- [15] D. Turkoglu, C. Alaca and C. Yildiz, "Compatible Maps and Compatible Maps of Types (α) and (β) in Intuitionistic Fuzzy Metric Spaces," *Demonstration Mathematica*, Vol. 39, No. 3, 2006, pp. 671-684.
- [16] B. Schweizer and A. Sklar, "Statistical Metric Spaces," *Pacific Journal Mathematic*, Vol. 10, 1960, pp. 314-334.
- [17] C. Alaca, I. Altun and D. Turkoglu, "On Compatible Mappings of Type (I) and Type (II) in Intuitionistic Fuzzy Metric Spaces," *Korean Mathematical Society*, Vol. 23, No. 3, 2008, pp. 427-446.
- [18] G. Jungck, "Compatible Mappings and Common Fixed Points," *International Journal of Mathematics and Mathematical Sciences*, Vol. 9, No. 4, 1986, pp. 771-779.
- [19] G. Jungck and B. E. Rhoades, "Fixed Point for Set Valued Functions without Continuity," *Indian Journal of Pure and Applied Mathematics*, Vol. 29, No. 3, 1998, pp. 227-238.